# Chapter 12

# Radiation I. Thermal radiation

## 12.1 The nature of electromagnetic radiation

The development of theories of light and radiation can be found in *Duncan*. Basically, two competing theories on the nature of light were advanced in the seventeenth century — the **corpuscular theory**, supported by Newton, regarded light as a stream of tiny particles. On the other hand, the **wave theory** proposed by the Dutch physicist Huygens around 1680 considered light to travel as waves. It was not until 1801 that *Thomas Young* obtained experimental evidence that light could produce wave effects (the famous Young's double slits experiment). The wave theory of light is often called the **classical theory** of electromagnetic radiation. In 1905 *Albert Einstein* suggested that the energy in light could be carried by particles (**photons**) whose energy depended on the wavelength of the light. An important aspect of the photon theory of light was that the energy of the photons should be **quantised**–that is, that photons can carry only discrete amounts of energy. This development led to the **quantum theory** of light and matter in general.

It is now considered that *either* the wave theory or the particle (quantum) theory of light can be used in a problem depending on the circumstances. A good example of the quantum theory being appropriate is the *photoelectric effect*, while the appearance of *diffraction fringes* when light passes through a narrow slit is a good example of the wave theory being more appropriate. This **wave-particle duality** nature of light is one of the enduring fascinations of the unification of classical and quantum physics that occurred at the beginning of the 20th century.

## 12.2 The electromagnetic spectrum

For any wave motion, the velocity of the wave is

$$v = f\lambda,$$

where $f$ is the frequency (units: $s^{-1}$) and $\lambda$ is the wavelength (units: m). For electromagnetic waves in a vacuum $v = 3 \times 10^8$ $ms^{-1}$.

The electromagnetic radiation (henceforth called just 'radiation') that we *see* as visible light constitutes only a tiny fraction of the total spectrum of radiation. Figure 12.1 shows the range of electromagnetic radiation together with some common designations from gamma rays to radio waves.

Visible light extends from about 400 nm (blue light) to about 900 nm (red light).

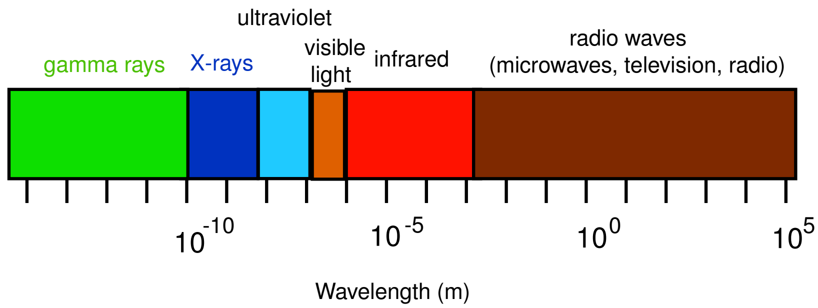


Figure 12.1: The electromagnetic spectrum.

## 12.3 Thermal radiation

All matter in the universe with a temperature greater than absolute zero emits electromagnetic radiation. Radiation arises from the atomic and molecular thermal vibrations that are going on all the time in gases, liquids and solids. Radiation can also arise from energy transitions of electrons in an atom or molecule, leading to radiation with a discrete wavelength (orange street lights, for example, are the result of emission by sodium vapour at only two discrete wavelengths - the sodium D-lines). The radiation emitted by a body as a result of its temperature is called **thermal radiation**.

Matter in the condensed phase (i.e., either solid or liquid) emits a continuous spectrum of thermal radiation. The details of this spectrum are almost independent of the particular material of which the body is composed, but they depend strongly on temperature. At ordinary temperatures most bodies are visible to us not by their **emitted light** but by the **reflected light**; if no light shines on them we cannot see them. Our common experience tells us that objects become visible by their emitted light only when they are heated to a high temperature, at which point they begin to glow. We also know from common experience that as we heat an object (for example a poker in a fire) the colour and intensity of the light changes as it gets hotter. A poker glows first red at moderate temperatures but then bright yellow and eventually an intense blue-white. This means that at increasing temperatures a body emits more thermal radiation and the wavelength of the most intense radiation decreases. It is no surprise then that at normal temperatures objects emit at wavelengths greater than those of visible light, thus objects become 'visible' only when viewed in the infrared.

The *thermal* radiative properties of bodies must be understood in order to describe the thermal state of the Earth and its atmosphere (including any additional tweaking that we call the greenhouse effect). The temperature of the Earth is determined by an equilibrium between its thermal radiation (mainly in the infrared, like most objects around us) and the thermal radiation from the sun (an incandescent 6000 °C). More on this later.

### 12.3.1 The Blackbody

The detailed spectrum of thermal radiation emitted by a body depends somewhat on the composition of the body. However, experiments show that there is one class of bodies that emits radiation of a universal character. These are called **blackbodies** and an understanding of their properties is fundamental to treating real bodies.

Blackbodies have surfaces that absorb all radiation incident upon them. The name is appropriate because they do not reflect any light and they appear black when temperatures are low enough that they don't emit any visible radiation themselves. *All* blackbodies at the same temperature emit radiation with the same spectrum. Note that although the word 'black' suggest we are dealing only with visible light. However, blackbodies, by definition, absorb *all* radiation of *all* wavelengths. Note also that they need not appear black! If we heat a blackbody it will begin to glow just like any other body–the term blackbody is really just a descriptive one highlighting the fact that they absorb all radiation incident upon them and therefore appear black at normal temperatures.

If a blackbody is in thermal equilibrium with its surroundings then it must *emit* as much thermal radiation as it *absorbs*. If this were not the case the body would heat up and no longer be in thermal equilibrium. The spectrum of the emitted thermal radiation is a function of wavelength and temperature only, and is shown in Figure 12.2. Note that whatever radiation is incident on a blackbody it will always emit with the blackbody spectrum appropriate to its temperature. If we shine just blue light on a blackbody it will absorb all of the blue light energy and re-emit the energy over all wavelengths, as in Figure 12.2. The total energy emitted over all wavelengths will be equal to the total energy of the incident blue light. The energy emitted by a radiating body is known as its **radiance**, which is the energy emitted per unit area of emitting surface. The radiance is a function only of temperature, and is given by

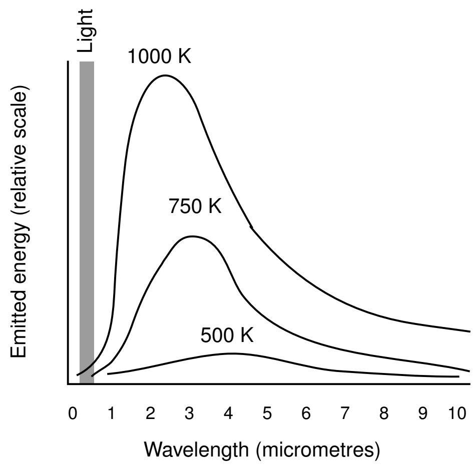


Figure 12.2: Blackbody spectra for three temperatures.

$$R_T = \sigma T^4. \quad (12.1)$$

This is **Stefan's law** and was first stated in this form in 1879. The constant, $\sigma$, is called the *Stefan-Boltzmann constant*.

Figure 12.2 also shows that the blackbody spectrum shifts towards higher frequencies as temperature increases (in accord with common experience). This is known as **Wien's displacement law**

$$\nu_{max} \propto T \quad (12.2)$$

where $\nu_{max}$ is the frequency at which $R_T$ has its maximum value for a particular temperature.

The German physicist Max Planck derived the following relationship for the spectrum of intensity emitted by a blackbody

$$B_\lambda(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(e^{\frac{hc}{\lambda kT}} - 1)}, \quad (12.3)$$

where $h$ is Planck's constant ($6.62 \times 10^{-34}$ J s$^{-1}$) and $c$ is the speed of light ($3 \times 10^8$ m s$^{-1}$). This is known as the **Planck or Blackbody spectrum**. In this equation, $B(\lambda, T)$ is the **radiant intensity** or **radiance** (W m$^{-2}$m$^{-1}$. Radiance is the energy emitted per square metre per unit wavelength per second.

The **Stefan-Boltzmann law**, given in Equation 12.1, was based on empirical evidence before the derivation of Equation 12.3 by Planck. The total flux emitted by a blackbody can be determined by integrating Equation 12.3 over the electromagnetic spectrum, giving

$$R_T = \frac{2k^4\pi^4T^4}{15h^3c^2} \quad (12.4)$$

### 12.3.2 Emissivity

Real substances are not perfect blackbody emitters. Rather, they emit a fraction of the radiation emitted by a blackbody. The radiance actually emitted by a body is

$$E_T = \epsilon\sigma T^4 \tag{12.5}$$

where $\epsilon$ is the **emissivity**. The emissivity is wavelength dependent. This means that a body may be a very efficient emitter at ultraviolet wavelengths, but a very poor one at infrared wavelengths. The emissivities of different surface types at typical infrared wavelengths are given in Table 12.1.

| Surface | Emissivity |
|---|---|
| Liquid water | 1.0 |
| Fresh snow | 0.99 |
| Old snow | 0.82 |
| Ice | 0.96 |
| Soil | 0.9-0.98 |
| Concrete | 0.71-0.9 |

Table 12.1: Emissivities of some common surfaces.

## 12.4 Radiative heating and cooling

The energy *emitted* by a blackbody of surface area $A$ at temperature $T$ is

$$E_{\text{emit}} = A\sigma T^4 \text{ W}.$$

The energy *absorbed* by a blackbody if the surroundings are at a temperature of $T_0$ is

$$E_{\text{absorb}} = A\sigma T_0^4 \text{ W}.$$

The difference between $E_{\text{emit}}$ and $E_{\text{absorb}}$ is the net heat transfer to or from the body, $q$:

$$q = A\sigma(T^4 - T_0^4).$$

This equation can be used to calculate the rate at which hot bodies cool down by emitting thermal radiation. A body will be in thermal equilibrium when it has the same temperature as the surroundings ($T = T_0$, such that $q = 0$). Note that a body loses heat by conduction at a rate proportional to the difference in temperature while a body losing heat by radiation does so at a rate proportional to $T^4$. This implies that radiative heat loss can be very efficient.

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EXAMPLE 13 *Calculate the rate at which a metal sphere of radius 10 cm at a temperature of 400 K cools when placed in a room at temperature 298 K. Assume the density of the sphere is $\rho = 8000$ kg m$^{-3}$ and that it has a specific heat capacity of $c = 400$ J kg$^{-1}$ K$^{-1}$.*

*First, let's calculate the rate of heat loss. The surface area of the sphere is $4\pi r^2$. The energy lost to the surroundings per second is*

$$q = 4\pi(10 \times 10^{-2})^2\sigma(400^4 - 298^4) \;=\; 126 \text{ W}.$$

*The sphere has a mass of $m = \frac{4}{3}\pi r^3 \times$ density. If the initial rate of temperature decrease is $\frac{dT}{dt}$ then*

$$q = mc\frac{dT}{dt} = \frac{4}{3}\pi(10 \times 10^{-2})^3\rho c\frac{dT}{dt} = \sigma 4\pi(10 \times 10^{-2})^2(400^4 - 298^4).$$

*The rate of change of temperature is therefore*

$$\frac{dT}{dt} = 0.0094 \text{ Ks}^{-1},$$

*or about 33 K per hour.*

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## 12.5 Radiative equilibrium

Equation 12.1 tells us how much energy is emitted from a blackbody per second per unit area of its surface. If the blackbody is in thermal equilibrium (it has constant temperature) then it must be absorbing as much radiation from its surroundings as it is emitting. Thus, at thermal equilibrium *absorption = emission.*

### 12.5.1 The temperature of the Earth

We can use this simple statement of radiative equilibrium to calculate the temperature of the Earth assuming that its only source of heat is from radiation emitted by the sun. The Earth absorbs about $1.2 \times 10^{17}$ W of energy from the sun, averaged over the Earth's surface. We can calculate the temperature of the Earth by equating this absorbed radiance with the radiance emitted by a blackbody Earth at temperature $T_{\text{Earth}}$. In radiative equilibrium:

$$1.2 \times 10^{17} = \sigma T_{\text{Earth}}^4 A_{\text{Earth}},$$

where $A_{\text{Earth}}$ is the surface area of the Earth. The radius of the Earth is about 6400 km, giving $A_{\text{Earth}} = 4\pi r_{\text{Earth}}^2 = 5.15 \times 10^{14}$ m$^2$. This gives $T_{\text{Earth}} = 253$ K, or -20 °C.

Figure 12.3 shows the blackbody spectrum of the sun (temperature 5780 K) and the Earth (temperature 256 K, close to what we just calculated). This demonstrates that the sun's energy is primarily in the visible part of the spectrum, while the Earth's radiance is mainly in the infrared (thus, the Earth does not glow in the dark!)

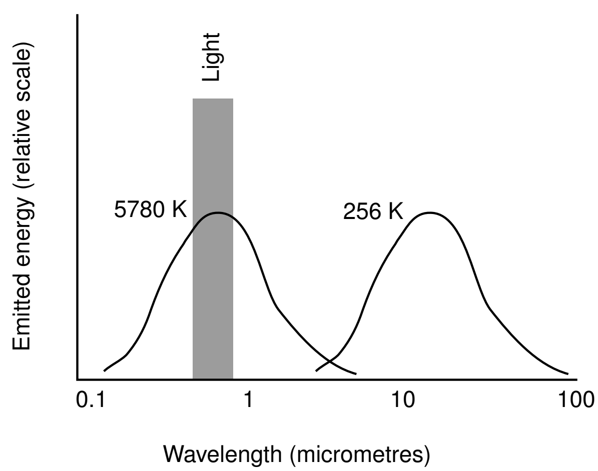


Figure 12.3: Blackbody spectra for the sun and the Earth. Note that the radiances are not to scale, since the total radiance of the sun is obviously much greater than that of the Earth.

The temperature calculated above is clearly not correct; the Earth's average temperature is significantly higher than -20 °C! The error in the calculation was the assumption that the Earth has no atmosphere. In reality, the gaseous atmosphere is a strong absorber of infrared radiation emitted by the Earth's surface. The atmosphere is relatively transparent to radiation incident upon it from the sun but relatively opaque to the infrared radiation re-emitted by the Earth.

## Questions for understanding

1. Explain what is meant by a blackbody
2. Define emissivity.
3. Calculate the temperature of the Earth if only 80% of the incident radiation were absorbed, rather than the assumed 100% used in the example in this lecture.
4. What can you say about the wavelength dependent emissivity of green grass?
5. Calculate the radiative heat loss through a 1 $m^2$ window assuming that the internal temperature is 20°C and the external temperature is -5°C. How does this compare with the conductive heat loss that you calculated in question 4 in lecture 6?
6. If a 100 W light bulb has a filament with a surface area of 4 $mm^2$ and the bulb is emitting light with 100% efficiency, what is the temperature of the filament? How could you have made a rough guess at this temperature given what you know about the temperature of the sun.