3.   PERMEABILITY

3.1   Theory

The *permeability* of a rock is a measure of the ease with which the rock will permit the passage of fluids.

The fundamental physical law which governs this is called the Navier-Stokes equation, and it is very complex. For the purposes of flow in rocks we can usually assume that the flow is laminar, and this assumption allows great simplification in the equations.

It should also be noted that the permeability to a single fluid is different to the permeability where more than one fluid phase is flowing. When there are two or more immiscible fluid phases flowing we use relative permeability, which will be introduced in this section, but covered in much more detail on the Formation Evaluation course later in the MSc.

The fluid flow through a cylindrical tube is expressed by Poiseuille's equation, which is a simplification of Navier-Stokes equation for the particular geometry, laminar flow, and uncompressible fluids. This equation can be written as

$$Q = \frac{\boldsymbol{p}\ r^4\ (P_i - P_o)}{8\ \boldsymbol{m}\ L}$$

(3.1)

where: Q = the flow rate (cm^3/s or m^3/s)
             r = the radius of the tube (cm or m)
             P_o = the outlet fluid pressure (dynes/cm^2 or Pa)
             P_i = the inlet fluid pressure (dynes/cm^2 or Pa)
             $\boldsymbol{m}$ = the dynamic viscosity of the fluid (poise or Pa.s)
             L = the length of the tube (cm or m)

About 150 years ago Darcy carried out simple experiments on packs of sand, and hence developed an empirical formula that remains the main permeability formula in use in the oil industry today. Darcy used the apparatus shown in Fig. 3.1, where he used a vertical sand pack through which water flowed under the influence of gravity while measuring the fluid pressures at the top and bottom of the pack by the heights of manometers. Here the difference in fluid pressures can be calculated from h_1–h_2 providing the density of the fluid is known. It has since been validated for most rock types and certain common fluids. Darcy's formula can be expressed as

$$Q = \frac{k\ A\ (P_i - P_o)}{\boldsymbol{m}\ L}$$

(3.2)

where: Q = the flow rate (cm^3/s or m^3/s)
             P_o = the outlet fluid pressure (dynes/cm^2 or Pa)
             P_i = the inlet fluid pressure (dynes/cm^2 or Pa)
             $\boldsymbol{m}$ = the dynamic viscosity of the fluid (poise or Pa.s)
             L = the length of the tube (cm or m)
             k = the permeability of the sample (darcy or m^2)
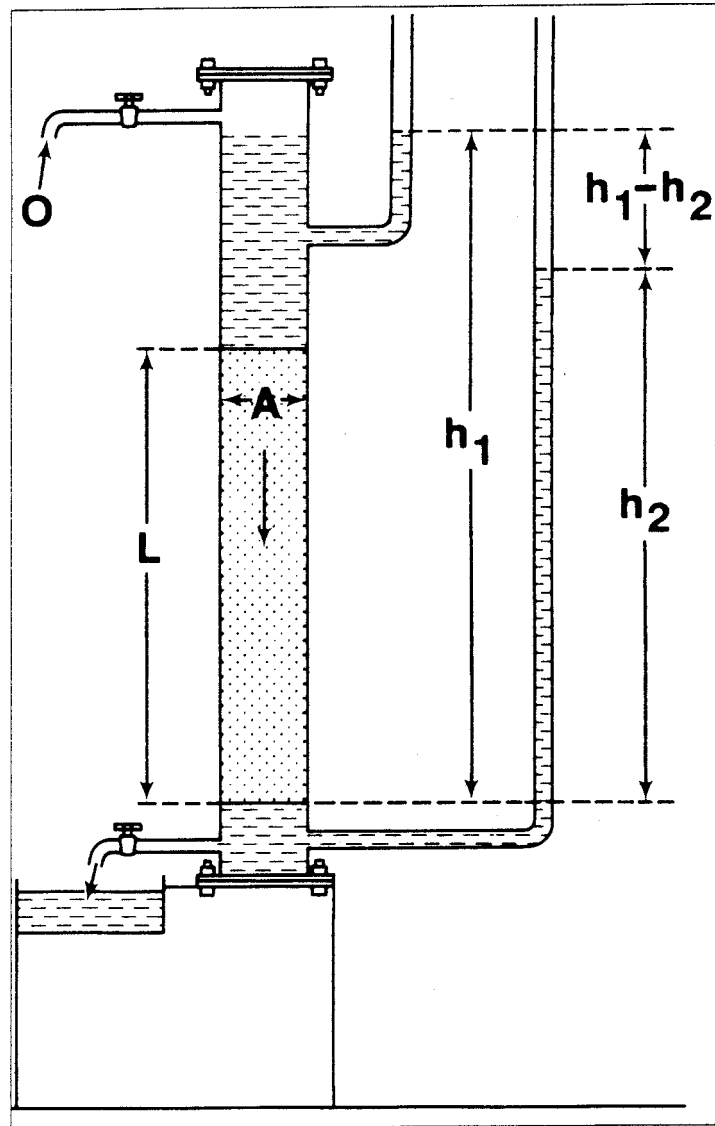 A = the area of the sample (cm^2 or m^2)

Figure 3.1 The Darcy apparatus (after Hubert, 1953).

Note, either the first set of quoted units can be used (c.g.s system) as in the oil industry, or the second set (S.I. units) as in academic research, but one must use the units consistently.

The units of permeability are the darcy, D, and m^2, where 1 D = 0.9869×10^{-12} m^2. One darcy is the permeability of a sample 1 cm long with a cross-sectional area of 1 cm^2, when a pressure difference of 1 dyne/cm^2 between the ends of the sample causes a fluid with a dynamic viscosity of 1 poise to flow at a rate of 1 cm^3/s (Fig. 3.2). In geological applications the darcy is commonly too large for practical purposes, so the millidarcy (mD) is used, where 1000 mD = 1D.
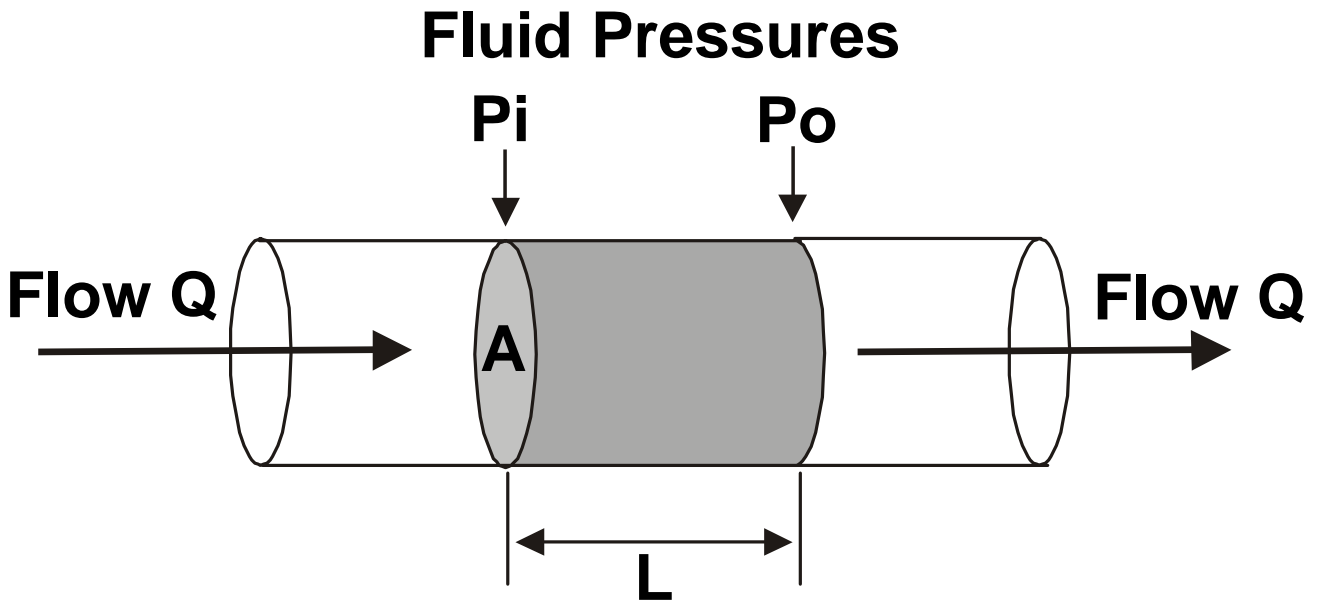
Fluid Pressures

Pi   Po

Figure 3.2 Permeability definition.

For practical calculations the following rearranged equations are used

For liquids (oil and water):

$$k = 1000 \frac{L}{A} \, \boldsymbol{m} \; Q \frac{1}{\left(P_o - P_i\right)} = 1000 \frac{L}{A} \, \boldsymbol{m} \frac{\Delta V}{\Delta T} \frac{1}{\left(P_o - P_i\right)} \tag{3.3}$$

where: $\boldsymbol{DV}$ = the volume of liquid flowed in time $\boldsymbol{DT}$ (cm^3)
$\boldsymbol{DT}$ = the time period over which flow is measured (s)
Q = the flow rate = $\boldsymbol{DV}/\boldsymbol{DT}$ (cm^3/s)
P_o = the outlet fluid pressure (atmospheres absolute, atma)
P_i = the inlet fluid pressure (atmospheres absolute, atma)
$\boldsymbol{m}$ = the dynamic viscosity of the fluid (centipoise, cP)
L = the length of the sample (cm)
k = the permeability of the sample (millidarcy, mD)
A = the area of the sample (cm^2).

For gasses (hydrocarbon gasses or nitrogen):

$$k = 2000 \frac{L}{A} \, \boldsymbol{m} \; Q \frac{P_{atm}}{\left(P_o^2 - P_i^2\right)} = 2000 \frac{L}{A} \, \boldsymbol{m} \frac{\Delta V}{\Delta T} \frac{P_{atm}}{\left(P_o^2 - P_i^2\right)} \qquad \grave{} \tag{3.4}$$

where: $\boldsymbol{DV}$ = the volume of gas flowed in time $\boldsymbol{DT}$ measured at atmospheric pressure (cm^3)
$\boldsymbol{DT}$ = the time period over which flow is measured (s)

Q = the flow rate = DV/DT (cm^3/s)
P_o = the outlet fluid pressure (atmospheres absolute, atma)
P_i = the inlet fluid pressure (atmospheres absolute, atma)
P_{atm} = The atmospheric pressure (atmospheres absolute, atms, =1)
m = the dynamic viscosity of the fluid (centipoise, cP)
L = the length of the sample (cm)
k = the permeability of the sample (millidarcy, mD)
A = the area of the sample (cm^2).

3.2 Controls on Permeability and the Range of Permeability Values in Nature

Intuitively, it is clear that permeability will depend on porosity; the higher the porosity the higher the permeability. However, permeability also depends upon the connectivity of the pore spaces, in order that a pathway for fluid flow is possible. The connectivity of the pores depends upon many factors including the size and shape of grains, the grain size distribution, and other factors such as the operation of capillary forces that depend upon the wetting properties of the rock. This is a complex subject that will be covered in more detail in the Formation Evaluation course later in the MSc.

However, we can make some generalizations if all other factors are held constant:

- The higher the porosity, the higher the permeability.
- The smaller the grains, the smaller the pores and pore throats, the lower the permeability.
- The smaller the grain size, the larger the exposed surface area to the flowing fluid, which leads to larger friction between the fluid and the rock, and hence lower permeability.

The permeability of rocks varies enormously, from 1 nanodarcy, nD (1×10^{-9} D) to 1 microdarcy, μD (1×10^{-6} D) for granites, shales and clays that form cap-rocks or compartmentalize a reservoir, to several darcies for extremely good reservoir rocks. In general a cut-off of 1 mD is applied to reservoir rocks, below which the rock is not considered as a reservoir rock unless unusual circumstances apply (e.g., it is a fractured reservoir). For reservoir rocks permeabilities can be classified as in Table 3.1 below.

Table 3.1 Reservoir permeability classification.

Permeability Value (mD)	Classification
<10	Fair
10 – 100	High
100 – 1000	Very High
>1000	Exceptional

3.3 Permeability Determination

Permeability is measured on cores in the laboratory by flowing a fluid of known viscosity through a core sample of known dimensions at a set rate, and measuring the pressure drop across the core, or by setting the fluid to flow at a set pressure difference, and measuring the flow rate produced.

At this point we must make a distinction between the use of gaseous fluids and the use of liquids. In the case of liquids the measurement is relatively straightforward as the requirement for laminar flow and incompressibility of the fluid are almost always met at surface geological conditions. If one wants to use gas as the fluid, as is commonly done in the industry, there are two complications:

Gas is a compressible fluid, hence if gas is flowing at the same mass per unit time through the core, it will actually be travelling more slowly when measured in volumes per time at the input (high pressure) end of the sample because it is compressed into a smaller volume, than at the output end (low pressure) where it expands. The equation used to calculate the permeability value from the measured parameters has to be modified to take the gas compression into account.

At low gas pressures, there can be very few molecules of gas occupying some of the smaller pores. If this happens, the laws that we are using breakdown, and their use causes an overestimation in the permeability. This is known as *gas slippage* or the *Klinkenberg Effect*. The problem becomes smaller as the pressure is increased because the gas is compressed and there are more gas molecules per unit volume, and does not arise in liquids because liquids are very much denser than gasses.

Gas slippage is corrected for by making permeability measurements with gas at multiple pressure differences and constructing a graph of the measured apparent permeability against the reciprocal of the mean pressure in the core. If the input gas pressure is P_i and the output pressure is P_o, then the permeability is plotted as a function of $1/P_{av} = 2/(P_i + P_o)$, as in Fig. 3.3. The points should now lie on a straight line, which intersects the *y*-axis at $1/P_{av} = 0$. This value is called the Klinkenberg permeability, and effectively represents the permeability at which the gas (which is near to a perfect gas) is compressed by infinite pressure and becomes a near perfect liquid. It is because of this that the klinkenberg permeability is often given the symbol k_L.

The klinkenberg permeability is very commonly used within the oil industry, and should approximate very well to the permeability of the sample measured with liquid flowing through it. It should be noted that the correction cannot be ignored, especially in tight rocks, as it can lead to corrections of up to 100%. In general, the correction is smaller for higher permeability rocks containing larger pores.
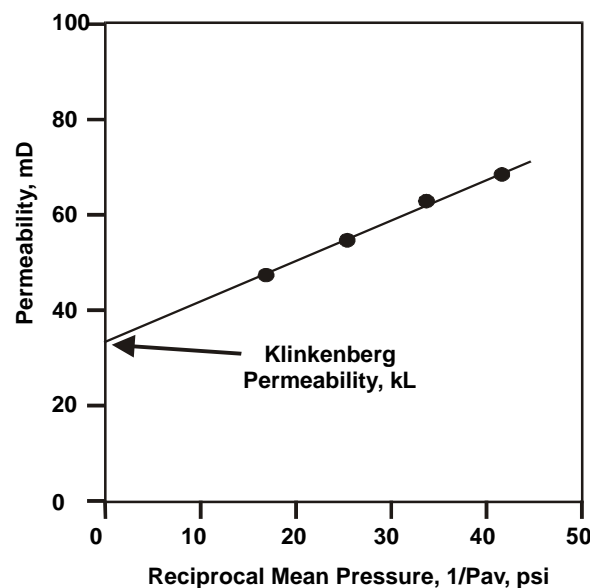

Figure 3.3 The klinkenberg correction plot.

Permeability is generally anisotropic in a rock, partly because of depositional effects, and partly because of the *in-situ* stress field in the crust. To account for this, permeability measurements are made both parallel to and perpendicular with bedding. The permeability perpendicular to bedding will be about a third to half of that parallel to bedding. Clearly this has implications for extracting oil from a reservoir, as oil would usually much rather travel laterally than vertically.

3.4   Relative Permeability

As indicated in Section 3.2, permeability depends upon many factors. Perhaps not surprisingly, one of those factors is the degree to which the available pore space is saturated with the flowing fluid. The pore space may not be completely saturated with one fluid but contain two or more. For, example, there may be, and generally is, both oil and water in the pores. What is more, they may both be flowing at different rates at the same time. Clearly, the individual permeabilities of each of the fluids will be different from each other and not the same as the permeability of the rock with a single fluid present. These permeabilities depend upon the rock properties, but also on the saturations, distributions, and properties of each of the fluids.

If the rock contains one fluid, the rock permeability is maximum, and this value is called the *absolute permeability*.

If there are two fluids present, the permeabilities of each fluid depend upon the saturation of each fluid, and can be plotted against the saturation of the fluid, as in Fig. 3.4. These are called *effective permeabilities*. Both effective permeabilities are always less than the absolute permeability of the rock and their sum is also always less than the absolute permeability of the rock. The individual effective permeabilities are most often expressed as a fraction of the absolute permeability of the rock to either of the two fluids when present at 100% saturation, and these are called *relative permeabilities*.

Hence, if 100% water occupies the rock, the absolute permeability to water is k_{aw}, and the same applies for 100% saturations with oil (k_{ao}), and gas (k_{ag}). If any two, or all three, of these fluids are present together in the rock at some partial saturation S_w, S_o and S_g, we can measure their *effective permeabilities*, which are k_{ew}, k_{eo}, and k_{eg}, which will all be less than their absolute values. We can define and calculate the relative permeability values by expressing the effective permeabilities as a fraction of some base permeability, which is arbitrary but usually the absolute permeability of one of the fluids present. For example, if we take k_{ao} as the base permeability, the relative permeabilities are:

$$k_{rw} = k_{ew}/k_{ao}$$
$$k_{rg} = k_{eg}/k_{ao,} \text{ and}$$
$$k_{ro} = k_{eo}/k_{ao.}$$

Note that the precise value of the relative permeabilities depends upon the base permeability with which they are calculated, and this should always be quoted whenever relative permeabilities are used.

Referring to Fig. 3.4, it can be seen that the effective and hence relative permeability of a given fluid decreases as the saturation of that fluid decreases, and that there is a threshold value of saturation of any given fluid that needs to be present before that fluid will move. This last point is on the one hand intuitive, as one would expect the need for sufficient of a given fluid to be present before a connected, flowable pathway could come into being, and on the other hand critical, because it implies that fluids become trapped (unmovable) in a rock when they are still present in significant amounts. In the figure, oil is immobile until its saturation is about 20%, indicating that we cannot produce from zones that

contain less than 20% oil, and that we will not be able to produce the last 20% of oil from zones which initially have higher oil saturations. This is known as the *residual oil saturation, S_or*.
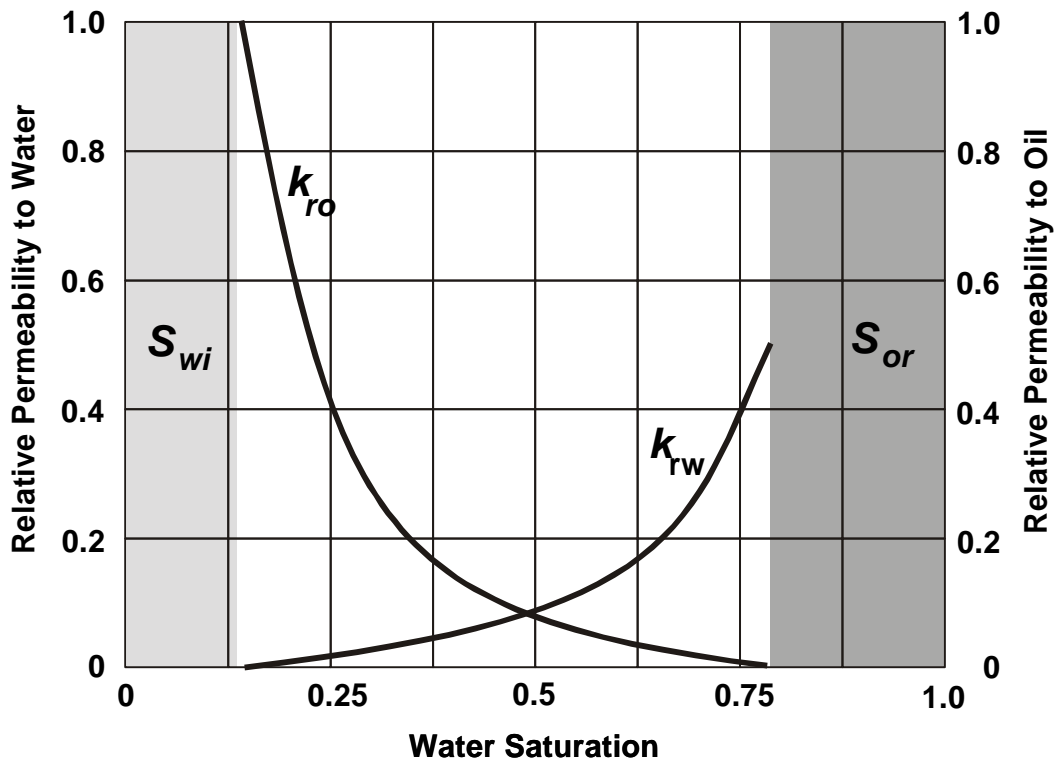

Figure 3.4 Relative permeability curves for an oil/water system that is water wet.

The same applied to the water, whose immobile fraction is termed the *irreducible water saturation, S_wi*, and gas, whose immobile fraction is termed the *trapped gas saturation, S_gt*.

There is a point on the plot where the curve intersect. Here the permeability to each fluid is the same, and both fluids are equally easily produced. As the oil saturation increases, the permeability to oil increases, and that to water decreases, and *vice versa*. Hence it is apparent that in oil reservoirs it is important to avoid the production of water as not only does it not make money, but an increasing *water cut* reduces the permeability of the reservoir to the oil, making oil more difficult to produce.

3.5.1 Well Productivity

As we have just seen, the permeability is an extremely important parameter because it controls the well's productivity. The productivity of a well is related both to the permeability and to the interval of the borehole open for production (i.e., the interval where the well casing is *perforated*).

For the arrangement shown in Fig. 3.5, we can write the flow equation

$$Q = \frac{k\,A\,(P_f - Pw)}{m\,L} = \frac{k\,2\,p\,\,r_w\,h\,(P_f - Pw)}{m\,L} = C\,k\,h \qquad \textbf{(3.3)}$$
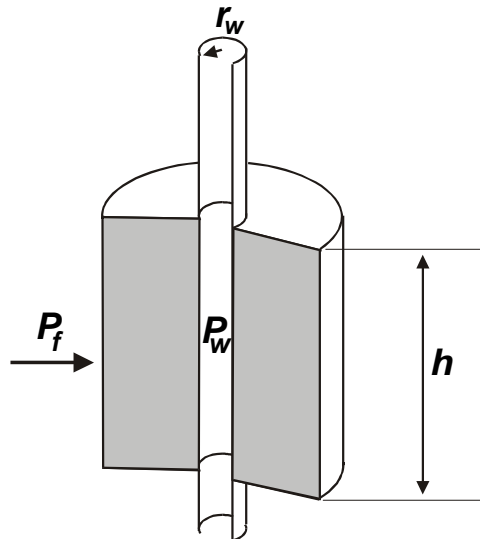
where, the internal area of the borehole has been substituted for the area *A*. It can be seen that for any instant in time, when the formation pressure P_f and the well pressure P_w are constant, and for constant fluid viscosity, **m**, the productivity of the well will be proportional to the permeability and the interval of production, *h*, with *C* being the coefficient of proportionality.

Figure 3.5   Well productivity.

3.6   PoroPerm Relationships

The most obvious control on permeability is porosity. This is because larger porosities mean that there are many more and broader pathways for fluid flow. Almost invariably, a plot of permeability (on a logarithmic scale) against porosity for a formation results in a clear trend with a degree of scatter associated with the other influences controlling the permeability. For the best results these poroperm cross-plots should be constructed for clearly defined lithologies or reservoir zones. If a cross-plot is constructed for a whole well with widely varying lithologies, the result is often a disappointing cloud of data in which the individual trends are not apparent. Figure 3.6 shows a poroperm cross-plot for a clean sandstone and a carbonate.
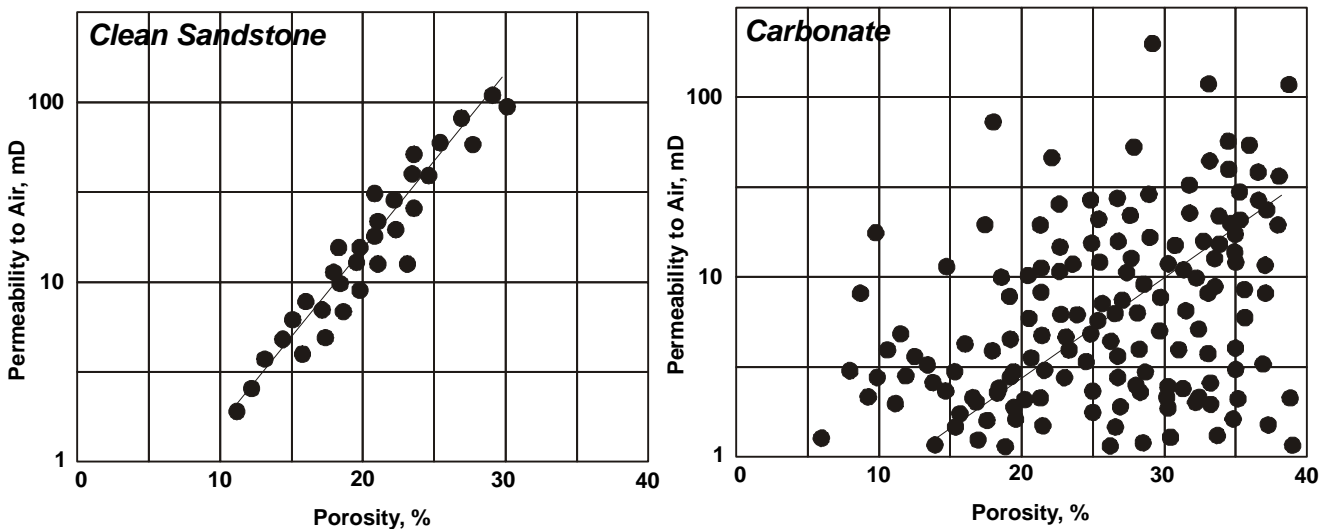

Figure 3.6 Typical poroperm cross-plots.

It is clear from this figure that the permeability of the sandstone is extremely well controlled by the porosity (although usually there is more scatter than in this figure), whereas the carbonate has a more diffuse cloud indicating that porosity has an influence, but there are other major factors controlling the permeability. In the case of carbonates (and some volcanic rocks such as pumice), there can exist high porosities that do not give rise to high permeabilities because the connectivity of the vugs that make up the pore spaces are poorly connected.

Poroperm trends for different lithologies can be plotted together, and form a map of poroperm relationships, as shown in Fig. 3.7.

It would be time consuming to describe the figure in detail, but interpretation is not difficult. For example, fractured rocks fall above the sandstones because their porosity (fracture porosity) is very low, yet these fractures form very connected networks that allow the efficient passage of fluids, and hence the permeability is high. Such permeability may be directional because of preferred orientations of the fractures. By comparison, clay cemented sandstones have high porosities, but the porosity is mainly in the form of micro-porosity filled with chemically and physically (capillary) bound water which is immobile. This porosity does not take place in fluid flow, so the permeability is low.
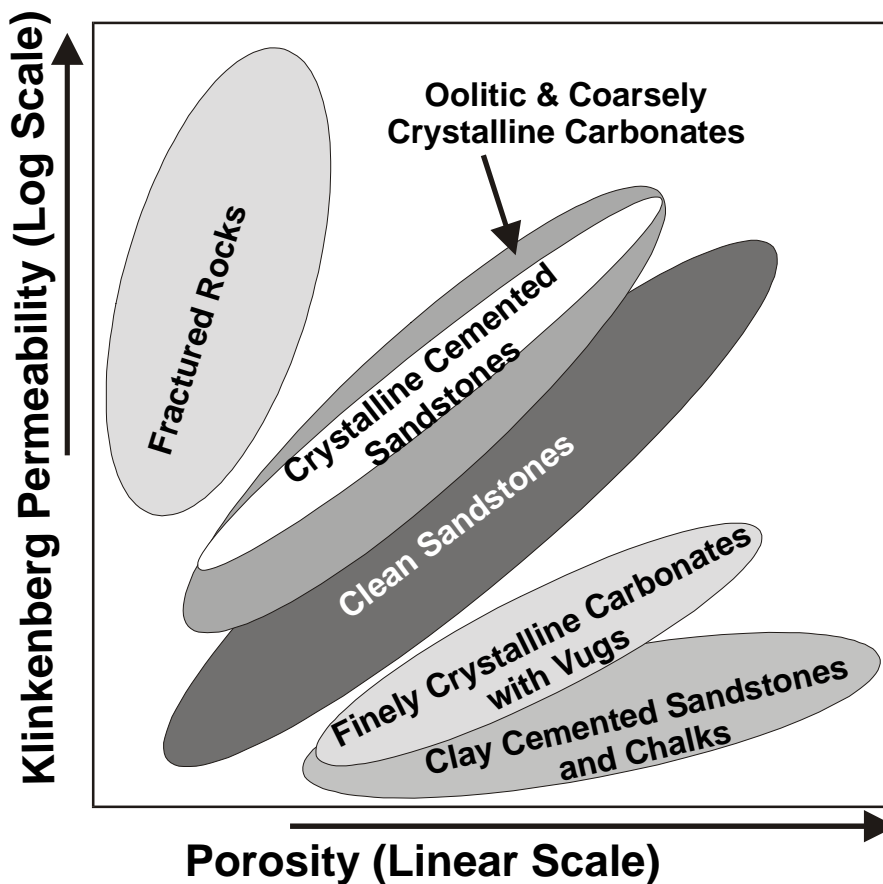

Figure 3.7 Poroperm relationships.

It might be expected that grain size also has some control on permeability. Figure 3.8 shows a poroperm cross-plot for a well in a carbonate reservoir where the grain size, porosity and permeability were measured for each core taken. Taking the data as a whole, there is little in the way of a clear trend. However, trends emerge when the individual grain size fractions are considered. Now it is clear that rocks with smaller grain sizes have smaller permeabilities than those with larger grain sizes. This is because smaller grain sizes produce smaller pores, and rather more importantly, smaller pore throats, which constrain the fluid flow more than larger grains which produce larger pore throats.

In summary, permeability:

- Depends upon porosity.
- Depends upon the connectivity of the flow paths in the rock.
- Depends, therefore, in a complex way upon the pore geometry of the rock.
- Is a directional quantity that can be affected by heterogeneous or directional properties of the pore geometry.
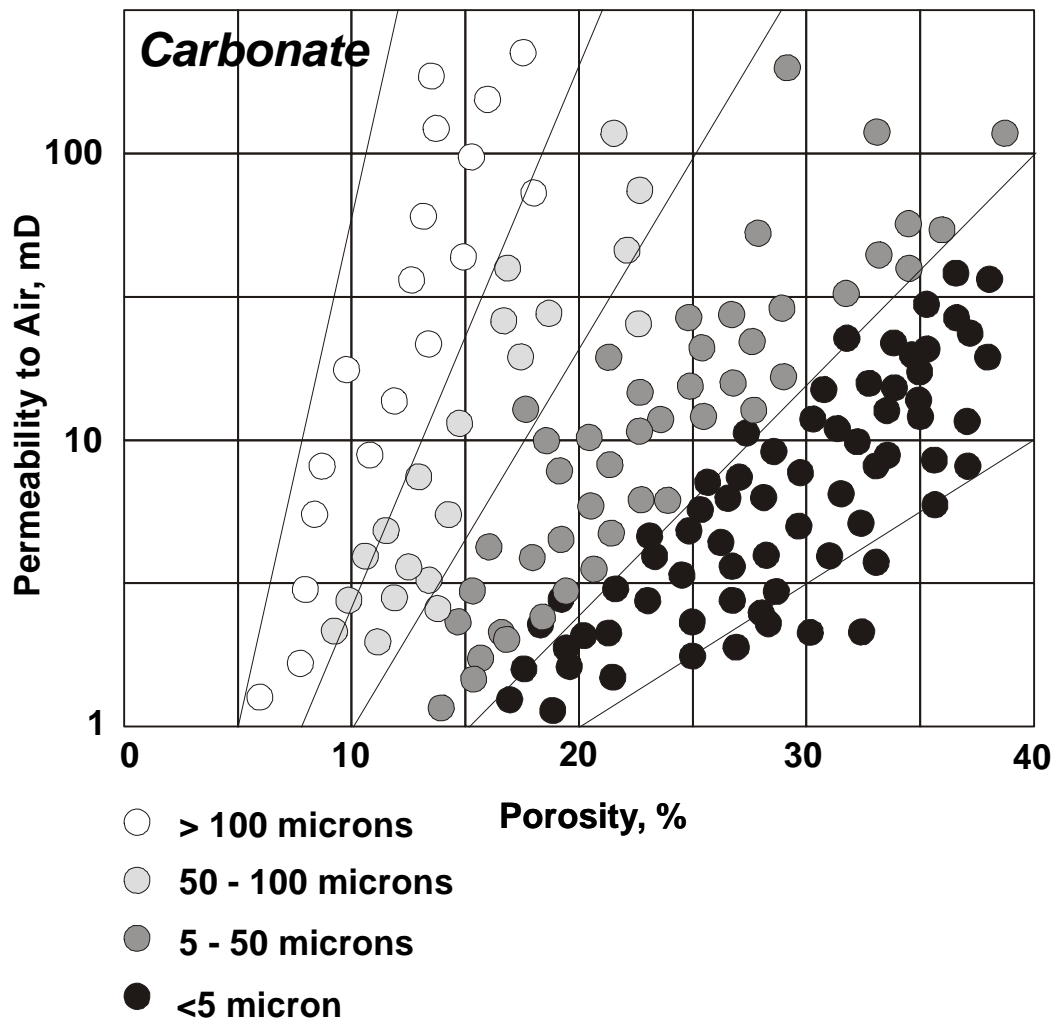-

Figure 3.8 Poroperm cross-plots and the influence of grain size.

3.7 Permeability Relationships

The complexity of the relationship between permeability and pore geometry has resulted in much research. No fundamental law linking the two has been found. Instead, we have a plethora of empirical approximations for calculating permeability, some of which are given in Table 3.2.

Table 3.2 Permeability relationships (modified from Grouping's list).

Name	Equation	Notes
Solution Channel	$k = 0.2 \times 10^8 \times d^2$	k = permeability (D) d = channel diameter (inches)
Fractures	$k = \dfrac{0.544 \times 10^8 \times w^3}{h}$	k = permeability (D) h = fracture width (inches) w = fracture aperture (inches)
Wyllie and Rose equations I	$k = \left(\dfrac{100\, f^{2.25}}{S_{wi}} \right)^2$	k = permeability (mD) f = porosity (fraction) S_{wi} = irreducible water saturation (fraction)
Wyllie and Rose equations II	$k = \left(\dfrac{100\, f^2 \left[1 - S_{wi} \right]}{S_{wi}} \right)^2$	k = permeability (mD) f = porosity (fraction) S_{wi} = irreducible water saturation (fraction)
Timur equation	$k = \dfrac{0.136\, f^{4.4}}{S_{wi}^2}$	k = permeability (mD) f = porosity (%) S_{wi} = irreducible water saturation (%)
Morris and Biggs equation	$k = \left(\dfrac{C\, f^3}{S_{wi}^2} \right)$	k = permeability (mD) f = porosity (fraction) S_{wi} = irreducible water saturation (fraction) C = constant; oil=250; gas=80
Slichter equation	$k = \dfrac{10.2\, d^2}{K_s}$	k = permeability (mD) d = median grain size (microns) K_s = packing correction; slope of line when plotting median grain size vs. permeability.
Kozeny-Carman equation	$k = \dfrac{c\, d^2\, f^3}{(1 - f)^2}$	k = permeability (mD) f = porosity (fraction) c = constant d = median grain size (microns)
Berg equation	$k = 8.4 \times 10^{-2} \times d^2\, f^{5.1}$	k = permeability (mD) f = porosity (fraction) d = median grain size (microns)
Van Baaren equation	$k = 10\, D_d^2\, f^{(3.64+m)}\, C^{-3.64}$	k = permeability (mD) f = porosity (fraction) D_d = modal grain size (microns) C = sorting index m = Archie cementation exponent.
RGPZ equation	$k = \dfrac{1000\, d^2\, f^{3m}}{4\, a\, m^2}$	k = permeability (mD) d = weighted geometric mean grain size (microns) ϕ = porosity (fraction) m = Archie cementation exponent. a = grain packing constant