

# What is the cementation exponent? A new interpretation

PAUL GLOVER, *Université Laval*

In 2003, 182 billion barrels of oil reserves worth about US \$4.5 trillion were discovered worldwide (Johnson et al., 2004). Moreover, between 1950 and 2002 the total volume of reserves discovered has run to more than 1500 billion barrels for oil and 7.5 trillion cubic feet for gas (Bentley, 2002). More than half of these resources have already been produced and have driven the global economy for the last 50 years.

While these hydrocarbon discoveries were aided by expertise in geology, geophysics, and engineering (as well as plain good luck!), all of the assessments of the volume of hydrocarbon reserves were made using data from petrophysical measurements together with a set of relationships that originated with Archie in 1942. Therefore, it would be difficult to overestimate the impact of either the petrophysical techniques or Archie's relationships on the worldwide economy.

Archie's laws link the electrical resistivity of a rock to its porosity, to the resistivity of the water that saturates its pores, and to the fractional saturation of the pore space with the water. They are used to calculate the hydrocarbon saturation of the reservoir rock from which the reserves are then calculated. Archie's laws contain two exponents,  $m$  and  $n$ , which Archie called the cementation exponent and the saturation exponent, respectively. The conductivity of the hydrocarbon-saturated rock is highly sensitive to changes in either exponent. Fortunately, the saturation exponent does not vary much ( $n=2\pm\frac{1}{2}$ ). However, the cementation exponent commonly takes values from just over 1 to around 5. Water and oil saturations calculated with Archie's equations are highly sensitive to this level of variability in the cementation exponent, but, thankfully, there are a number of ways in which the cementation exponent can be calculated with precision (e.g., Tiab and Donaldson, 1994).

Despite the importance of the cementation exponent, few petrophysicists, commercial or academic, are able to describe its real physical meaning. Some authors (e.g., Ellis and Singer) even relegate the cementation exponent to the status of a "fitting parameter" in an empirical relationship. While this position was probably valid 20 years ago, Archie's laws and their parameters have a healthier theoretical foundation today. The purpose of this paper is to investigate the elusive physical meaning of the cementation exponent.

## Traditional interpretations

Archie began by naming the ratio of the resistivity of the rock  $\rho_o$  to that of the pore water  $\rho_w$  the resistivity formation factor

$$F = \frac{\rho_o}{\rho_w} \quad (1)$$

The term formation factor was used because it was approximately constant for any given formation. The formation factor varies from unity,  $F = 1$ , which represents the case where  $\rho_o = \rho_w$  (i.e., when  $\phi \rightarrow 1$ ), and increases as the porosity decreases, with  $F \rightarrow \infty$  as  $\phi \rightarrow 0$ . The formation factor can be

less than unity, but only when the rock matrix is less resistive than the pore water, and this is extremely rare.

The first series of experiments carried out by Archie led him to the conclusion that the formation factor depends upon porosity in the form of an inverse power law

$$F = \phi^{-m} \quad (2)$$

with an exponent  $m$ . He called the exponent the cementation exponent (factor or index) because he believed it to be related to the degree of cementation of the rock fabric. This constitutes the first attempt to understand the meaning of the cementation exponent, however qualitative it may be. It is clear from the form of the equation that higher values of  $m$  make the formation factor more sensitive to changes in the rock's porosity and are associated with higher values of tortuosity (lower connectivity) (Ellis and Singer, 2007).

The range of values for the cementation exponent is relatively small. A value of  $m = 1$  is not observed for real rocks, and represents a porous medium composed of a bundle of capillary tubes which cross the sample in a straight line. Rocks with a low porosity but a well developed fracture network sometimes have cementation exponents that approach unity because the network has flow paths that are fairly direct. Here we get the first taste that the cementation exponent has something to do with the connectedness of the pore and fracture network (where, for the time being, connectedness is considered to be a qualitative term for the general availability of pathways for transport).

A cementation exponent equal to 1.5 represents the analytical solution for the case where the rock is composed of perfect spheres (Sen et al., 1981; Mendelson and Cohen, 1982). In fact,  $m=1$  and  $m=1.5$  were until recently the only two cases where an analytically derived value of the cementation exponent was known. A series of papers from 2004 onwards has shown that Archie's law can be derived by applying continuum percolation theory to fractal porous media (e.g., Ewing and Hunt, 2006). Most porous arenaceous sediments have cementation exponents between 1.5 and 2.5 (Glover et al., 1997). Values higher than 2.5, and as high as 5, are generally found in carbonates where the pore space is less well connected (Tiab and Donaldson, 1994). In general, the value of the cementation exponent increases as the degree of connectedness of the pore network diminishes, which rather supports it being called the cementation exponent.

Incidentally, values of the cementation exponent less than unity are possible, and arise particularly in the modified Archie's law for two conducting phases (Glover et al., 2000). In this model there is an exponent representing each phase, and if the exponent related to the conducting pore fluid is greater than unity, the other exponent, which represents the conducting rock matrix, takes a value less than unity. It is as if, in a 3D porous medium, there is only so much connectedness possible. If a certain high degree of connectedness is taken by

one phase, represented by a low cementation exponent, the other phase must have a high cementation exponent that represents its lower degree of connectedness. The link between the two cementation exponents  $m$  and  $p$  is given by

$$p = \frac{\log(1 - \phi^m)}{\log(1 - \phi)} \quad (3)$$

and the modified form of the Archie's first law is given by

$$\rho_o = \rho_w \phi^{-m} + \rho_s (1 - \phi)^{-p} \quad (4)$$

where the fluid phase has a resistivity  $\rho_w$  and completely occupies the porosity  $\phi$ , and the solid conducting phase has a resistivity  $\rho_s$  and occupies the rest of the rock.

One way of approaching the meaning of the cementation exponent would surely be to examine the way it is usually measured in the laboratory (Tiab and Donaldson, 1994). Here the logarithm to any base of both sides of Equation 2 is taken, then the equation is rearranged to give

$$m = - \frac{\log(F)}{\log(\phi)} \quad (5)$$

This equation may be used to calculate the cementation exponent for any sample where the resistivity formation factor and porosity have been measured, but it is more common and more acceptable to calculate the cementation exponent for a suite of samples of the same facies by calculating the negative gradient of the graph of  $\log(F)$  as a function of  $\log(\phi)$ . This latter approach nullifies the effect of variability within the dataset providing that care is taken to only include measurements from samples that are clearly of the same electrofacies, and hence provides a value more useful for reservoir modeling. Equation 5 provides another "definition" of the cementation exponent, but unfortunately not one that gives any insight into its physical meaning.

Archie's first law is often expressed by combining the first two equations as

$$\rho_o = a \rho_w \phi^{-m} \quad (6)$$

This is wrong! It implies that a beaker of the saturating fluid (i.e.,  $\phi \rightarrow 1$ ) does not have a resistivity  $\rho_w$  as it should, but a resistivity  $a \times \rho_w$  instead, which is clearly in error. Reports which contain values of the constant  $a$  that are anything other than exactly unity are the result of sloppy thinking and the mindless application of curve fitting programs. The real equation is

$$\rho_o = \rho_w \phi^{-m} \quad (7)$$

Here we can see that  $\rho_o = \rho_w$  as  $\phi \rightarrow 1$ . It is also worth noting that  $\rho_o = \rho_w$  for any value of porosity providing  $m = 0$ . Although  $m = 0$  does not fall within the useful range for reservoir rocks, this result is a further indication that low values of cementation exponent represent good connectedness, and in the limit (i.e., when  $m \rightarrow 0$ ) the connectedness of the rock is optimal because the porosity is 100%.

Finally, there is another parameter related to the resistivity formation factor that is useful; that of the tortuosity. Electrical tortuosity is most often written as

$$\tau = F \phi \quad (8)$$

which, although it is a useful form, does not clearly show its origins. It is possible to rewrite Equation 2 as

$$F = \phi \phi^{1-m} \quad (9)$$

where it can be recognized that the resistivity formation factor has a contribution arising purely from the presence of a scalar porosity, and another which arises from the way that porosity (whatever it is) is arranged. This latter contribution is called the tortuosity,  $\tau$ . Hence, the tortuosity can also be expressed as

$$\tau = \phi^{1-m} \quad (10)$$

### A new interpretation

I have taken a slightly different approach to the understanding of the physical meaning of the cementation exponent.

First I use conductivity in place of resistivity. This may, at first sight, seem a trivial difference, but it is a difference of approach from which everything else flows. It is necessary to understand that we tend to use resistivity rather than conductivity for calculating water and hydrocarbon saturations for purely historical reasons. Resistivity is what was generally measured when electrical drilling began in 1927 by the Schlumberger brothers—it was then only natural for Archie to use resistivity rather than conductivity when he published his relationships.

However, conductivity, the inverse of resistivity, has a better pedigree when it comes down to physics. For example, the conductivity of a solution is given by the density of charge carriers  $n$  multiplied by their charge  $q$  and by their mobility  $\beta$  according to  $\sigma = n\beta q$  where the conductivity  $\sigma$  is the coefficient of proportionality between the applied electrical field  $\mathbf{E} = -\text{grad}V$ , where  $V$  is the electrical potential, and the resulting electrical current density  $\mathbf{J}$

$$\mathbf{J} = \sigma \mathbf{E} = -\sigma \text{grad}V \quad (11)$$

In physics, only then is the resistivity defined as the inverse of the conductivity.

Having adopted the conductivity regime, it is possible to define a conductivity formation factor

$$G \equiv \frac{\sigma_o}{\sigma_w} = \frac{1}{F} \quad (12)$$

Like the resistivity formation factor, the conductivity formation factor is also approximately constant for a given facies. The conductivity formation factor varies from zero, which represents the case where  $\sigma_o = 0$  (i.e., when  $\phi \rightarrow 0$ ) and increases as the porosity increases, with  $G \rightarrow 1$  (i.e.,  $\sigma_o = \sigma_w$ ) as  $\phi \rightarrow 1$ . We are now in a position to attempt a physical interpretation of this relationship. The conductivity formation factor is the conductivity of the rock normalized to the conductivity of the saturating fluid. In other words, the conductivity formation factor describes the conductivity of a mixture of solid matrix and fluid relative to a sample composed only of the fluid. The conductivity formation factor is, in effect, a dilution factor where the pore fluid is diluted by rock grains. It is a dilution factor where the conductivity of the rock is not only affected by the replacement of a given volume of fluid with the same volume of solid matrix, but also by the arrangement of the resulting solid matrix. Hence, the conductivity formation factor is also a measure of the availability of pathways for electrical transport. The conductivity formation factor is, in fact, a measure of the connectedness of the pore and fracture network of a sample.

Hence we will define  $G$  as the *connectedness* of a porous medium.

In the conductivity regime, the equivalent forms of Equations 2, 5, and 7 are

$$G = \phi^m \quad (13)$$

$$m = \frac{\log(G)}{\log(\phi)} \quad (14)$$

$$\sigma_o = \sigma_w \phi^m \quad (15)$$

but these relationships do not directly lend themselves to any better physical interpretation of the cementation exponent than their equivalents in the resistivity regime.

However, if we define a connectivity for the pore network  $\chi$ , that is the inverse of the tortuosity  $\tau$  according to

$$\chi \equiv \frac{1}{\tau} \quad (16)$$

it is possible to write (from Equation 8) that

$$\chi = \phi^{m-1} \quad (17)$$

This provides an alternative interpretation for the connectedness (conductivity formation factor) as the product of the porosity of the rock and its connectivity

$$G = \phi^m = \phi \phi^{m-1} = \phi \chi \quad (18)$$

This relationship elegantly expresses the idea that the general connectedness of a rock is in part due to the amount of pore volume available for electrical conduction (represented by the porosity) and the way that that porosity is arranged in three dimensions (represented by the connectivity).

The next step is to examine the differential characteristics of the connectedness (conductivity formation factor), especially its rate of change with porosity

$$\frac{dG}{d\phi} = m\phi^{m-1} = m\chi \quad (19)$$

A further differentiation with respect to the connectivity provides

$$m = \frac{d}{d\chi} \left( \frac{dG}{d\phi} \right) \quad (20)$$

Equation 20 leads to a very reasonable physical meaning for the cementation exponent: The cementation exponent is the sensitivity to changing connectivity of the sensitivity to changing porosity of the connectedness (conductivity formation factor). Written in words it is a mouthful, but physically quite elegant. The connectedness (conductivity formation factor) describes how the conductivity of 100% fluid is modified by the presence of solid non-conducting grains. *The cementation exponent then takes account of how that modification varies as a function of porosity and of the connectivity of the matrix.*

Furthermore, by combining Equations 18 and 19 we get

$$\frac{dG}{d\phi} = \frac{d(\chi\phi)}{d\phi} = m\chi \quad (21)$$

and then by differentiating the resulting product, it is possible to obtain

$$\frac{d\chi}{d\phi} = \frac{\chi(m-1)}{\phi} \quad (22)$$

That is to say, the rate of change of connectivity of a rock with porosity depends upon its initial connectivity, the cementation exponent and the initial porosity. Whereas a large initial connectivity will augment the change in connectivity, a large porosity has the effect of diminishing the change in connectivity. In other words, if you add a crack to an otherwise low-porosity rock the connectivity will change more abruptly than adding the same crack to a rock that already has a high porosity. Or for connectivity, if you add a link between pores or cracks in a well connected pore network, the result is that the network increases its connectivity more than if the same link were added to a low connectivity network.

**In summary**

The main conceptual steps in this paper are summarized as:

- Connectedness of a porous medium is defined as the availability of pathways for transport.
- Connectedness is the inverse of the formation resistivity factor and may also be called the conductivity formation factor.
- Porosity is defined as the fractional amount of pore space.
- Connectivity is defined as the measure of how the pore space is arranged.
- Connectivity is given by  $\chi = \phi^{m-1}$  and depends upon the porosity and the cementation exponent  $m$ .
- Connectedness is given by  $G = \phi\chi$  and depends upon the amount of pore space (porosity) and the arrangement of the pore space (connectivity).
- The rate of change of connectedness with porosity  $\frac{dG}{d\phi} = m\chi$  depends upon the connectivity  $\chi$  and the cementation exponent  $m$ .
- The rate of change of the connectedness with porosity and connectivity is equal to the cementation exponent,  $\frac{d}{d\chi} \left( \frac{dG}{d\phi} \right) = m$ .
- Hence, the cementation exponent is interpreted as being the rate of change of the connectedness with porosity and connectivity,  $m = \frac{d^2G}{d\chi d\phi}$ .

**Suggested reading.** “The electrical resistivity log as an aid in determining some reservoir characteristics” by Archie (*Transactions of the American Institute of Mechanical Engineers*, 1942). “Global oil & gas depletion: an overview” by Bentley (*Energy Policy*, 2002). *Well Logging for Earth Scientists* by Ellis and Singer (Springer, 2007). “Dependence of the electrical conductivity on saturation in real porous media” by Ewing and Hunt (*Vadose Zone Journal*, 2006). “Damage of saturated rocks undergoing triaxial deformation using complex electrical conductivity measurements: Experimental results” by Glover et al. (*Physics and Chemistry of the Earth*, 1997). “A modified Archie’s law for two conducting phases” by Glover et al. (*EPSL*, 2000). *Strategic Significance of America’s Oil Shale Resource, Volume I, Assessment of Strategic Issues* by Johnson et al. (U.S. Department of Energy, 2004). “The effects of grain anisotropy on the electrical properties of sedimentary rocks” by Mendelson and Cohen (*GEOPHYSICS*, 1982). “Self similar model for sedimentary rocks with application to the dielectric constant of fused glass beads” by Sen et al. (*GEOPHYSICS*, 1981). *Petrophysics: Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties* by Tiab and Donaldson (Gulf Publishing Company, 1996). **TLE**

Corresponding author: paul.glover@ggl.ulaval.ca