Numerical simulation of collapsing volcanic columns with particles of two sizes

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Abstract. A three-phase thermofluid-dynamic model was employed to simulate the behavior of collapsing volcanic columns and related pyroclastic flows. The model accounts for the mechanical and thermal nonequilibrium between a gas phase and two solid phases representative of particles of two different sizes. The gas phase has two components: hot water vapor leaving the vent and atmospheric air. Collisions between particles of the same size were accounted by a solids elasticity modulus, whereas a semiempirical correlation was employed to account for particle-particle interactions between particles of different sizes. The gas phase turbulence was modeled by a turbulent subgrid scale model. The partial differential equations of conservation of mass, momentum, and energy were solved numerically, by a finite difference scheme, on an axisymmetric physical domain for different granulometric compositions at the vent. Simulations were limited to particles of few hundreds microns, and therefore to dilute flows, in order to maintain a reasonable computational load. Results show the formation of the initial vertical jet, column collapse, building of a pyroclastic fountain followed by the generation of a radially spreading pyroclastic flow, and the development of convective instabilities from the upper layer of the flow which lead to the formation of coignimbritic or phoenix clouds. The analysis of the spatial and temporal distributions of the two solid phases in the different parts of the domain shows nonequilibrium effects between them and allow us to quantify important emplacement processes as pyroclast sedimentation and ash dispersion. In particular, the importance of coupling effects between the two solid phases leads to relevant differences between the behavior of columns with one or two solid phases. A significant influence of the granulometric composition was observed on the pyroclastic flow runout, flow thickness, and particle distribution in the flow and phoenix cloud. The results from simulations appear to be qualitatively in agreement with simple laboratory experiments and field observations.

Introduction

Collapsing volcanic columns and pyroclastic flows are very common phenomena in explosive eruptions. For this kind of eruption the magmatic material leaving the crater is composed by a mixture of molten and solid pyroclasts in a continuous gas phase. Such a mixture is the product of gas exsolution and magma fragmenta-

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sedimentation and sorting inside the flow, and generation of coignimbrite or phoenix columns from the upper layer of the flow [Sparks and Wilson, 1976; Walker, 1981; Wright et al., 1980; Dobran et al., 1993].

In the past decades numerous efforts have been made in the classification and quantification of deposits resulting from explosive eruptions [Walker, 1973; Sparks, 1976; Sparks and Wilson, 1976; Fisher, 1979; Wright et al., 1980; Walker, 1981; Fisher and Schmincke, 1984; Cas and Wright, 1987]. Such data included, for instance, the study of deposit morphology, textural features, and composition. Particular attention was paid to the determination of the pyroclasts size distribution and to its implication for flow dynamics. Plinian eruptions were shown to eject particles ranging from few microns to some meters, with about 40 wt % less than 600 μm and up to 90 wt % less than 5 cm [Sparks and Wilson, 1976]. Large collapsing columns, which produce ignimbrites, have poorly sorted tephra with a fine ash (smaller than 65 μm) content up to 85 wt % [Walker, 1981]. Coignimbrite columns collect the finer portion of ashes from the flow and very often produce thick layers in the stratigraphic sequence. Furthermore, pyroclastic flows were divided into several categories as a function of their physical and rheological properties, first of all their particle concentration and distribution. However, despite many geological studies, no quantitative classification of these phenomena is available and pyroclastic flows remain, from several points of view, unknown.

In the last 10 years, both physical modeling and laboratory experiments have been employed to further investigate the physical behavior of explosive eruptions [Neri and Macedonio, 1995]. Researchers at Los Alamos National Laboratory [Wohletz et al., 1984; Valentine and Wohletz, 1989a, b; Wohletz and Valentine, 1990; Valentine et al., 1991, 1992] applied, for the first time, transient, two-dimensional, two-phase flow models to describe large-scale behavior of plinian, collapsing, and caldera-forming blast eruptions. The modeled solid phase accounted for only one particle size, whereas the gas phase consisted of one chemical component. These works investigated the effects of the vent physical properties of the mixture on the collapse conditions, the influence of caldera rim on the propagation of pyroclastic flows, and several sources of column unsteadiness such as material recirculation into the main jet leaving the vent and the ventward wind generated by the ascending convective plumes. Horn [1989] extended such models by the consideration of large noncolliding particles in addition to a dusty gas phase. More recently, Dobran et al. [1993] presented a further extension of the Valentine and Wohletz's [1989a] model, by the implementation of two chemical components in the gas phase (water vapor and air) and the application of the kinetic theory for granular flows in the description of the particle-particle interaction [Savage, 1988; Ding and Gidaspow, 1990]. From parametric studies [Neri and Dobran, 1994] and applications of the model to past eruptions [Giordano and Dobran, 1994; Dobran et al., 1994] it was possible to better describe the formation of phoenix columns from pyroclastic flows and to give evidence of further complex and nonintuitive behaviors of these phenomena. Among the most important are the fountain height and pyroclastic flow mass flow rate pulsations, partial collapse of the columns at the transition between the plinian and collapsing columns, and the nonlinear relationship between particle size and flow runout. Finally, Dobran [1993] highlighted the major tasks in order to reach a proper integration of pyroclastic dispersion modeling with geological, geophysical, and petrological data.

The laboratory experiments of Carey et al. [1988], Woods and Caulfield [1992], and Woods and Bursik [1994], aimed at a reproduction of some aspects of the real eruptive dynamics, are qualitatively consistent with the simulation results obtained with the above described models. All these theoretical and experimental works demonstrated the very complex nature of these kinds of phenomena and highlighted the nonlinearity of thermofluid-dynamics processes. As a consequence, it is extremely dangerous to directly associate stratigraphic evidence with simple emplacement mechanisms and eruption sequences. From this point of view, numerical simulation represents a powerful tool to properly investigate the real physics of these complex phenomena.

The purpose of this paper is to present an extension of the above mentioned models able to describe pyroclasts by two distinct solid phases representative of two different particle sizes. The model describes, on an axisymmetric physical domain, the temporal behavior of a three-phase mixture, constituted by a gas phase of water vapor and air and two solid particulate phases. Simulations were performed both with one and two particle sizes in order to highlight the effect of the granulometric composition on the investigated phenomena. Model results allow us to quantify the nonequilibrium processes in the various regions of the flow and to give more realistic estimates of the main physical processes that characterize collapsing columns and pyroclastic flows.

Description of Governing Equations and Numerical Scheme

The numerical simulations of pyroclastic flows carried out in this work are based on a thermo-fluid-dynamic model where a set of partial differential equations account for mass, momentum, and energy balance of one gas phase and two solid particle phases representative of two different grain sizes. The model is based on previous works by Gidaspow and Ettehadieh [1983], Shih et al. [1987], Syamlal and O'Brien [1988], and Bouillard et al. [1989a] who developed multiphase models for the simulation of the thermofluid-dynamics of fluidized bed combustors and various industrial equipments. Such models were appropriately modified to take account of the specific properties of the volcanological system [Dobran et al., 1993].

The gas phase is assumed to be composed of two chemical components such as the water vapor leaving the crater, and the atmospheric air (considered as a sin-
gle chemical component). At the beginning of the simulation, the atmosphere is stratified in pressure and temperature (standard atmosphere) in order to account for the real atmospheric properties. The two solid phases represent two types of particles, with different density and diameter, that are commonly present in a pyroclastic flow.

Mass, Momentum, and Energy Transport Equations

The mass conservation equations for the two gas chemical components (air and water vapor) and for the two particulate phases are

\[
\frac{\partial}{\partial t} \epsilon_g \rho_g + \nabla \cdot (\epsilon_g \rho_g \mathbf{v}_g) = 0
\]

(1)

\[
\frac{\partial}{\partial t} \epsilon_a \rho_a y_a + \nabla \cdot (\epsilon_a \rho_a y_a \mathbf{v}_a) = 0
\]

(2)

\[
\frac{\partial}{\partial t} \epsilon_w \rho_w y_w + \nabla \cdot (\epsilon_w \rho_w y_w \mathbf{v}_w) = 0
\]

(3)

\[
\frac{\partial}{\partial t} \epsilon_k \rho_k + \nabla \cdot (\epsilon_k \rho_k \mathbf{v}_k) = 0, \quad k = 1, 2
\]

(4)

with

\[
y_a + y_w = 1; \quad \epsilon_g + \sum_{k=1}^{2} \epsilon_k = 1
\]

(5)

where \(\epsilon\) is the volumetric fraction, \(\rho\) is the mass density, \(\mathbf{v}\) is the velocity vector, \(y\) is the mass fraction of the two gas chemical components, \(t\) is time, and the subscripts \(g\), \(a\), \(w\), and \(k\) (\(k = 1, 2\)) refer to the gas phase, air, water vapor, and two solid phases, respectively.

The momentum balance equations for the three phases can be written as follows:

\[
\frac{\partial}{\partial t} \epsilon_g \rho_g \mathbf{v}_g + \nabla \cdot (\epsilon_g \rho_g \mathbf{v}_g \mathbf{v}_g) = -\epsilon_g \nabla P_g + \nabla T_g
\]

\[+ \epsilon_g \rho_g \mathbf{g} + \sum_{k=1}^{2} D_{g,k} (\mathbf{v}_k - \mathbf{v}_g)
\]

(6)

\[
\frac{\partial}{\partial t} \epsilon_k \rho_k \mathbf{v}_k + \nabla \cdot (\epsilon_k \rho_k \mathbf{v}_k \mathbf{v}_k) = -\epsilon_k \nabla P_k + \nabla T_k
\]

\[+ \epsilon_k \rho_k \mathbf{g} - D_{g,k} (\mathbf{v}_k - \mathbf{v}_g) + D_{k,j} (\mathbf{v}_j - \mathbf{v}_k), \quad k, j = 1, 2; \quad j \neq k
\]

(7)

where \(P_g\) is the gas pressure, \(T\) is the stress tensor, \(g\) is the gravitational body force, \(D_{g,k}\) is the gas-solids drag coefficient, and \(D_{k,j}\) is the drag coefficient between the particulate phases \(k\) and \(j\) (\(k, j = 1, 2; \quad j \neq k\)).

The energy balance equations for the three phases, in terms of enthalpy, can be written as

\[
\frac{\partial}{\partial t} \epsilon_g \rho_g h_g + \nabla \cdot (\epsilon_g \rho_g h_g \mathbf{v}_g) = \epsilon_g \left( \frac{\partial P_g}{\partial t} + \mathbf{v}_g \cdot \nabla P_g \right)
\]

\[+ \nabla \cdot (k_g \epsilon_g \nabla T_g) + \sum_{k=1}^{2} Q_k (T_k - T_g)
\]

(8)

\[
\frac{\partial}{\partial t} \epsilon_k \rho_k h_k + \nabla \cdot (\epsilon_k \rho_k h_k \mathbf{v}_k) = \nabla \cdot (k_k \epsilon_k \nabla T_k)
\]

\[- Q_k (T_k - T_g), \quad k = 1, 2
\]

(9)

where \(h\) is the enthalpy, \(T\) is the temperature, \(k\) is the thermal conductivity, assumed to represent an effective value when the flow is turbulent, and \(Q_k\) (\(k = 1, 2\)) are the heat transfer coefficients between the gas and the two particulate phases. Heat exchange between the two solid phases as well as viscous dissipation effects and radiation heat are ignored since they are of minor importance in comparison with convection, conduction, and gas-particles heat exchange [Valentine and Wohletz, 1989a; Dobran et al. 1993].

Constitutive Equations

The conservation equations (1)-(9) are closed by appropriate constitutive equations as the equations of state, the stress tensors, and the interfacial drag and heat coefficients between the different phases.

Equation of state and physical properties. We assume that the gas phase obeys the ideal gas law

\[
\rho_g = \frac{P_g}{RT_g}
\]

(10)

where \(R\) is the gas constant of a mixture of water vapor and dry air. The two particulate phases are assumed to be incompressible, that is, with constant density.

The temperatures of the three phases are determined from their enthalpy

\[
T_g = \frac{h_g}{C_p g}, \quad T_k = \frac{h_k}{C_p k}, \quad k = 1, 2
\]

(11)

where the specific heat of pyroclasts \(C_p k\) (\(k = 1, 2\)) is assumed constant and that of gas is determined from

\[
C_p g = y_w C_p w + y_a C_p a
\]

(12)

Table 1 summarizes the mean values of the physical properties and parameters which were employed in the simulations reported in the paper.

Gas and solids stress tensors The gas phase stress tensor is modeled by a turbulent subgrid scale model using an effective viscosity \(\mu_{ge}\) [Fan et al., 1985; Dobran et al., 1993] where

\[
T_g = 2\epsilon_g \mu_{ge} \tau_g
\]

(13)

with

\[
\tau_g = \frac{1}{2} (\nabla \mathbf{v}_g + (\nabla \mathbf{v}_g)^T) - \frac{1}{3} (\nabla \cdot \mathbf{v}_g) I \]

\[= \tilde{\tau}_g - \frac{1}{3} (\nabla \cdot \mathbf{v}_g) I
\]

(14)

\[
\mu_{ge} = \mu_g + \mu_{gt} = \mu_g + 0.01 \Delta r \Delta z \rho_g (2tr(\tilde{\tau}_g \cdot \tilde{\tau}_g))^{1/2}
\]

(15)

where \(\mu_g\) is the gas molecular viscosity and \(\Delta r\) and \(\Delta z\) indicate the radial and vertical dimensions of the computational cell. The effective gas viscosity includes contributions from the molecular and turbulent effects of the gas phase but ignores the modulating effect on
Table 1. Physical Properties and Parameters Employed in the Simulations
Reported in the Paper

<table>
<thead>
<tr>
<th>Property/Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_a ), Pa s</td>
<td>( 1.75 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \mu_a ), Pa s</td>
<td>( 10^{-5} )</td>
</tr>
<tr>
<td>( k_a ), W/m K</td>
<td>0.05</td>
</tr>
<tr>
<td>( k_w ), W/m K</td>
<td>0.26</td>
</tr>
<tr>
<td>( k_k (k = 1, 2) ), W/m K</td>
<td>2.2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>1.0</td>
</tr>
<tr>
<td>( C_{pa} ), m(^2)/s(^2) K</td>
<td>( (6 + 0.002 T_g - 0.3 \times 10^{-6} T_g^2) \frac{4.18 \times 10^3}{28.964} )</td>
</tr>
<tr>
<td>( C_{pw} ), m(^2)/s(^2) K</td>
<td>( (8 + 0.0015 T_g + 0.13 \times 10^{-5} T_g) \frac{4.18 \times 10^3}{18} )</td>
</tr>
<tr>
<td>( C_{pk} (k = 1, 2) ), m(^2)/s(^2) K</td>
<td>( 1.3 \times 10^3 )</td>
</tr>
</tbody>
</table>

the gas turbulence due to the presence of particles in the flow field [Crowe, 1982; Adeniji-Fashola and Chen, 1990; Dobran et al., 1993].

A general formulation of the solid stress tensor that accounts for the void fraction, velocity flow fields, pressure, and all the other local physical properties of a multiphase mixture does not exist today [Gidaspow, 1994]. In the present study the effective particulate phases stress tensor is composed of a viscous and coulombic portion and can be expressed by

\[
T_k = \tau_{v,k} - \tau_{c,k} I, \quad k = 1, 2 \tag{16}
\]

where the viscous tensor \( \tau_{v,k} \) is

\[
\tau_{v,k} = 2 \epsilon_k \mu_k \left[ \frac{1}{2} (\nabla v_k + (\nabla v_k)^T) - \frac{1}{3} (\nabla \cdot v_k) I \right], \quad k = 1, 2 \tag{17}
\]

and the coulombic component \( \tau_{c,k} \) is defined by

\[
- \partial \tau_{c,k} = \epsilon_k G(\epsilon_g) \partial \epsilon_g, \quad k = 1, 2 \tag{18}
\]

with

\[
G(\epsilon_g) = -10^{-8} \epsilon_g^{1.43} \quad \text{N/m}^2 \tag{19}
\]

where \( G(\epsilon_g) \) represents the solids elastic modulus and is analogous to the stress-strain relationship in solid mechanics. About the viscous component of the stress tensor, several values of the solids viscosity, \( \mu_s \), were employed in order to estimate its contribution. For values of solids viscosity ranging from 0.01 to 0.1 Pa s, employed in various studies with similar particles size and volumetric concentrations [Gidaspow et al., 1988; Bouillard et al., 1989b; Gidaspow, 1994], no appreciable effect was observed in the results. Therefore, as previously predicted by other authors [Bouillard et al., 1991], this term was neglected in the reported simulations. As far as the coulombic stress it should be noted that only a normal repulsive component is considered in the study in order to avoid impossible low values of void fraction and above all to make the numerical problem well posed [Gidaspow and Etchadie, 1983]. Expressions of the solid elastic modulus have been proposed, on an experimental basis, by several authors [Bouillard et al., 1989a, 1991; Gidaspow, 1994]. Here we adopted the correlation (19) [Gidaspow, 1994], but very similar results were obtained with different expressions. This is principally due to the fact that the term containing \( G(\epsilon_g) \) becomes significant at very low void fractions that are not reached in the dilute flows of our simulations.

Drag and heat interphase coefficients. The drag coefficient between the gas and the \( k \)th \((k = 1, 2)\) particulate phase, \( D_{g,k} \), was expressed by well-established semiempirical correlations. In the dilute region \((\epsilon_g \geq 0.8)\), the coefficient is obtained by correction of the standard drag function for a single sphere [Bird et al., 1990], whereas for the dense region \((\epsilon_g < 0.8)\) the well-known Ergun’s equation [Ergun, 1952] was used. Their expressions are for \( \epsilon_g \geq 0.8 \):

\[
D_{g,k} = \frac{3}{4} C_{d,k} \frac{\epsilon_g \rho_g \left| \frac{v_g - v_k}{d_k} \right|^{\epsilon_g - 2.7}}{d_k}, \quad k = 1, 2 \tag{20}
\]

where the drag coefficient \( C_{d,k} \) is given by

\[
C_{d,k} = \frac{24}{Re_k} \left[ 1 + 0.15 \Re_k^{0.667} \right], \quad Re_k < 1000 \tag{21}
\]

\[
C_{d,k} = 0.44, \quad Re_k \geq 1000 \tag{22}
\]

and for \( \epsilon_g < 0.8 \):

\[
D_{g,k} = \frac{150 \epsilon_g^2 \mu_g}{\epsilon_g d_k^2} + 1.75 \frac{\epsilon_g \rho_g \left| \frac{v_g - v_k}{d_k} \right|}{d_k} \left( \epsilon_g - 1 \right), \quad k = 1, 2 \tag{23}
\]

where the Reynolds numbers for the two phases are expressed by

\[
Re_k = \frac{\epsilon_g \rho_g d_k \left| \frac{v_g - v_k}{\mu_g} \right|}{d_k}, \quad k = 1, 2 \tag{24}
\]

particularly important is the interaction between particles of different size that results in the consideration of a particle-particle drag coefficient between the two solid phases. The importance of this contribution was suggested by several authors [Soo, 1967; Nakamura and Capes, 1976; Arastooopour et al., 1982; Gidaspow et al., 1986] who derived theoretical and experimental studies.
where Reynolds numbers are defined as heat transfer coefficients $Q_k$ by

$$Q_k = N_{ui} \beta_k \varepsilon_k$$

and for $e < 0.8$:

$$N_{uk} = (2 + 0.16Re_k^{0.67}), \quad Re_k < 200$$

$$N_{uk} = 8.2Re_k^{0.6}, \quad 200 < Re_k < 1000$$

$$N_{uk} = 1.06Re_k^{0.67}, \quad Re_k > 1000, \quad k = 1, 2$$

for $e > 0.8$:

$$N_{uk} = (2 + 0.11Re_k), \quad Re_k < 200$$

$$N_{uk} = 0.12(2Re_k^{1.83})^{0.83}, \quad 200 < Re_k < 1000$$

$$N_{uk} = 0.61Re_k^{0.67}, \quad Re_k > 1000, \quad k = 1, 2$$

where Reynolds numbers are defined as

$$Re_k = \frac{\rho_s d_k |v_s - v_k|}{\mu_g}$$

and the $k$th ($k = 1, 2$) Nusselt number is related to the heat transfer coefficient $Q_k$ by

$$Q_k = N_{uk} \frac{6k_s \varepsilon_k}{d_k^2}, \quad k = 1, 2$$

Finally, it should be noted, as discussed above, that the model ignores the water vapor condensation effect in the atmosphere. Simulation results suggest that this effect occurs after several minutes from the beginning of the eruptions and particularly with high water vapor content and large particle sizes due to the less effective heat transfer between the phases. At that time the simulations were stopped.

Initial and Boundary Conditions and Solution Procedure

The solution of the above described model equations was performed on a two-dimensional axisymmetric domain with appropriate initial and boundary conditions, as shown in Figure 1. The physical domain involves a radial extent $L$ and a vertical extent $H$, whereas the computational domain includes an additional external frame to specify the boundary conditions. The vent, of diameter $D_v$, is located in the lower left-hand corner of the computational grid. Initially, a standard atmosphere vertically stratified in pressure and temperature was considered all over the domain. The vent pressure, velocities, volumetric fractions, and temperatures of the three phases were assumed to be fixed and constant. Furthermore, in order to simplify the analysis, the three phases were considered in thermal and mechanical equilibrium at the vent. At the symmetry axis, $r = 0$, the radial gradients of all dependent variables were set to zero. At the ground boundary ($z = 0$), no mass and heat transfer are allowed, whereas the no-slip condition is assumed for the gas and solids velocity. At the upper and right outlet boundaries the mass, momentum, and energy fluxes are assumed to be continuous and correspond to free outflow and inflow of the gas-pyroclastic mixtures. Simulations were stopped when large inflow currents occurred in the domain due to their nonrealistic description.

The conservation and constitutive equations were numerically solved for $P$, $\varepsilon_p$, $\varepsilon_k$, $v_s$, $v_k$, $h_a$, and $h_k$ with $k = 1, 2$ by a finite difference algorithm [Harlow and Amsden, 1975]. The numerical scheme treats continuum equations and pressure and drag terms in the momentum equations implicitly, whereas the solid stress terms are solved explicitly. The final finite difference equation for pressure is solved, for each computational cell, by a combination of iterative methods in order to satisfy the prescribed mass residual. Finally, the solution of the energy balance equations for the enthalpies of the three phases is fully explicit.

Due to the explicit time differencing, the numerical stability places a limitation on the time step size. Accuracy considerations and computational limits also impose restrictions on the cell size of the computational grid. Very fine cell sizes provide more accurate results but demand very small time steps and of course very long computer times. In the performed simulations different nonuniform grid sizes, computational domains, and time steps were employed. Parametric studies were made on all these computational variables in order to guarantee an adequate resolution of the large-scale fluid dynamics and to optimize computer time [Dobran et al., 1993]. All simulations presented in this work were performed with a time step of 0.025 s and with a minimum cell size of 10 m.

Results

Several simulations were performed in order to investigate the main effects of the particle size distribution at the vent on the large-scale behavior of collapsing columns and pyroclastic flows. In this section results from only three selected simulations are presented due to the large amount of information contained in each one. However, the reported large-scale effects were observed in all the performed simulations and therefore can be considered representative of a general trend. In the following, results are essentially presented in order to discuss the effects of the multiphase formulation of the solid particles, whereas we refer to previous works about the effects of vent conditions [Neri and Dobran, 1994] and computational parameters [Dobran et al., 1993].
A summary of the vent conditions of the three simulations is shown in Table 2. All the selected simulations refer to a vent diameter of 100 m, an exit velocity of the mixture of 120 m/s, a temperature of 1200 K, a water vapor content of 0.8 wt %, and a total particle volumetric fraction of 0.01. Furthermore, since the crater geometry was not considered in the simulations, the eruptive jet was considered balanced with the local atmospheric pressure [Kieffer and Sturtevant, 1984]. The selected particle sizes for the simulations were 10 and 200 μm. Such a choice was based on volcanological as well as on physical modeling considerations. Stratigraphic studies clearly indicate that 10- and 200-μm particles represent an important contribution to the total particle distribution [Sparks, 1976; Walker, 1981]. Furthermore, experimental and theoretical studies give evidence of a very different behavior of these two particle sizes in fluidized systems [Wilson, 1980, 1984; Gidaspow, 1994; Kunii and Levelspie, 1995]. As reported above, in order to simplify the analysis, the three phases were also considered in thermal and mechanical equilibrium at the vent and the two particulate phases were assumed to have the same microscopic density. The last assumption appears to be consistent with fluidization studies that indicate in the particle size rather than in the particle density the main parameter controlling the behavior of a gas-particle system [Kunii and Levelspie, 1995]. Under these assumptions, the three simulations investigate the effects of different distributions of the vent volumetric fraction of the two particulate phases. On the one side, simulations A1 and A2 consider all particles of the same size and therefore they model the mixture as a two-phase. On the other side, simulation A3 distributes on two distinct solid phases the total particulate content of the mixture considering 50% of particle of 10 μm and 50% of particle of 200 μm.

The three simulations were performed with the same physical and computational parameters and therefore all the differences can be attributed to the different granulometric vent compositions of the mixture. In detail, they were performed on a nonuniform grid extending 10 km in radial direction and 3 km in vertical direction. In the radial direction, the cell size started with 10 m and thereafter slowly increased until reaching a cell width of 120 m, whereas vertically the cell size started with 10 m and increased until 100 m. Before presenting the behavior of simulation A3, in which the two solid phases are contemporarily present, it is important to...
brieﬂy describe the simulations A1 and A2 in order to better quantify their difference and therefore to highlight the importance of a multiphase formulation of the solid phase.

Eruption A1

This eruption is characterized by the presence of just one solid phase of particle of 10-μm size. Figure 2 illustrates the distribution of the particulate phase volumetric fraction in the atmosphere at different times from the beginning of the eruption. Figure 3 shows, at the same times, the distribution of the water vapor volumetric fraction. After 30 s from the beginning of the eruption, the volcanic jet leaving the vent has lost its vertical thrust and, at about 800 m height, starts its collapse. The collapsed portion of the column subsequently forms a radially spreading pyroclastic ﬂow that at 120 s has reached 2.5 km from the vent. At the same time, a hot water vapor plume above the fountain entrains some particles and continues to ascend toward the upper layers of the atmosphere. At about 300 s and 3 km from the vent (not shown in the ﬁgure), hot water vapor-rich gas and particles begin to separate from the pyroclastic ﬂows forming a phoenix column. At greater times (420 s) the phoenix column continues to grow generating a large column that tends to join the ascending plume above the fountain. The generation of the phoenix column is principally due to the diminishing radial momentum of the ﬂow that favors the development of the buoyancy forces and therefore the rupture of the unstable density gradient in the gas phase [Dobran et al., 1993]. At this time the pyroclastic ﬂow head has reached about 5.2 km from the vent. At 600 s, the two ascending plumes are joined and a cloud with particle volumetric fraction of about 10^-5 and water vapor volumetric fraction of about 10^-4 occupies a large part of the investigated physical domain. The ﬂow head is practically stopped at 5.2 km with no signiﬁcant front advance in the last 2 min.

The most relevant feature of this column is the strong mechanical and thermal coupling between the gas and solid phases. This is essentially due to the very small particle size that causes large drag forces and heat exchange between the two phases. This favors also the formation of quite thick pyroclastic ﬂows with a very low tendency to segregate particles. The formation of the phoenix column is also very effective and characterized by a high volumetric content of particles. The phoenix column also appears to strongly slow down the pyroclastic ﬂow limiting its maximum runout.

Eruption A2

Eruption A2 is characterized again by the presence of particles of one size, but now the particle size is 200 μm. Figures 4 and 5 illustrate the particle and water vapor volumetric fraction distribution, respectively. The dimensions of the reported domain and the contours levels of the two ﬁgures are the same as Figures 2 and 3. The timewise behavior of this column is very similar to that of simulation A1 during the ﬁrst 2 min of the eruption. After about 120 s the pyroclastic ﬂow head has reached about 2.5 km from the vent, and above the fountain a hot low particle concentration plume rises up. After about 420 s the pyroclastic ﬂow head has reached 6 km from the vent while the hot water vapor leaves the pyroclastic ﬂow and generates an ascending hot plume. Unlike simulation A1, simulation A2 does not show any evidence of particles following water vapor during the phoenix column ascent. The same effect can be observed at 600 s, when the pyroclastic ﬂow has reached 7 km from the vent and the water vapor has formed a large ascending plume of hot gas at about 3 km from the vent (see Figure 5).

As previously discussed by Dobran et al. [1994], Neri and Dobran [1994], and Giordano and Dobran [1994], simulations performed with a particle size of about 100-200 μm appear to maximize the pyroclastic ﬂow runout. For particles of 10 μm the runout appears to be limited by the low density of the ﬂow and by the effective generation of the phoenix column. For particles larger than about 200 μm the high sedimentation rate limits again the pyroclastic ﬂow runout and requires the employment of a computational grid with a higher resolution. The ﬁgures also show that the ﬂow is thinner than that associated with eruption A1, and because the phoenix cloud has a volumetric ash content less than 10^-10, it probably would not be visible to a distant observer (data from dust storms and aerosols show that the observation limit of a dust cloud of 20-μm particle size can be taken as ϵ = 10^-5 when observed from a distance of about 1 km and that a smaller visibility is expected for larger particles [Chepil and Woodruff, 1957; Horn, 1989; Willeke and Baron, 1993]). It should be also noted how the particle size of the ﬂow affects the generation and dispersion of the phoenix column. By comparison of the frames at 420 s of Figures 3 and 5, a slower formation of the water vapor-rich column for eruption A2 can be observed. Also extremely relevant is the difference in the ﬂow runout that results 2 km higher for simulation A2 after 10 min from the beginning of the eruption.

Table 2. Geometry and Flow Conditions of Gas and Two Solid Phases at Volcanic Vent

<table>
<thead>
<tr>
<th></th>
<th>Dv, m</th>
<th>Tg, K</th>
<th>vg, m/s</th>
<th>Yuv, wt %</th>
<th>dμ1, μm</th>
<th>dμ2, μm</th>
<th>ϵ1,υ</th>
<th>ϵ2,υ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>100</td>
<td>1200</td>
<td>120</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>A2</td>
<td>100</td>
<td>1200</td>
<td>120</td>
<td>0.8</td>
<td>10</td>
<td>200</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>A3</td>
<td>100</td>
<td>1200</td>
<td>120</td>
<td>0.8</td>
<td>10</td>
<td>200</td>
<td>0.005</td>
<td>0.005</td>
</tr>
</tbody>
</table>

At the vent the phases are assumed to be in thermal and mechanical equilibrium (P₀ = 0.1 MPa, ρp₁ = ρp₂ = 2300 kg/m³, vg, = Vpl,v = Vp₂,v = Vv, Tg,v = Tp₁,v = Tp₂,v = Tv).
**Eruption A3**

This eruption is characterized by the contemporary presence of two solid phases representative of particles of two different sizes. The particle volumetric distribution at the vent consists of 50% of particles of 10 μm, and 50% of particles of 200 μm.

Figures 6 and 7 illustrate the temporal distribution of the particle volumetric fraction for the two solid phases. The timewise evolution of this eruption is similar, in the first few minutes, to those of simulations A1 and A2 reported above. During this time, the two solid phases are strongly coupled and high concentration regions are common to the two particulate phases. At about 300 s (not shown in the figures) an unstable density gradient in the flow causes the formation of a phoenix column that is clearly visible at 420 s. At this time, the flow head is 5.6 km from the vent and the phoenix column is

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**Figure 2.** Distribution of particulate volumetric fraction in the atmosphere at 30, 120, 420, and 600 s of eruption A1. The contour levels shown are the exponents to the base 10 and, beginning from the outer or distant from the vent region, correspond to -10, -8, -6, -5, -4, -3, -2, and -1.
more than 2 km high. After 600 s, the flow has reached about 6.5 km from the vent and the phoenix cloud has joined to the ascending plumes above the fountain forming a larger convective cloud. The distribution of the water vapor is very similar to the distribution of the fine (10 \( \mu \)m) particles and is not shown here. The 200-\( \mu \)m particles are present in the phoenix column with a concentration of about \( 10^{-8} \), while the concentration of 10 \( \mu \)m particles inside the column is much larger, about \( 10^{-4} \); 10-\( \mu \)m particles can be found, with quite large concentrations (\( 10^{-5} \)), in the more distal region of the flow, that is, at about 6.5 km from the vent after about 600 s.

Figures 8 and 9 show the volumetric fraction distribution inside the pyroclastic flow at different times of the 10-\( \mu \)m and 200-\( \mu \)m particle, respectively. The represented physical domain is 7 km wide and 0.1 km high in order to give special attention to the vertical gradients.

**Figure 3.** Distribution of the water vapor volumetric fraction in the atmosphere at 30, 120, 420, and 600 s of eruption A1. The contour levels shown are the exponents to the base 10 and, beginning from the outer or distant from the vent region, correspond to $-10, -8, -6, -5, -4, -3, -2, -1, 0$. 
of the investigated variables. The figures demonstrate the different behavior of the two phases. After 180 s, 10-μm particles are distributed on a 60- to 80-m-high flow with a concentration ranging from $10^{-3}$ at the ground to $10^{-10}$ at the upper surface. On the other side, 200-μm particles are concentrated in a thinner basal part of the flow, about 40 m high, and therefore show a larger vertical concentration gradient. Furthermore, the inner contour line, corresponding to $10^{-3}$, covers a wider portion of the flow for 200-μm particle size with respect to 10-μm particles. At 300 s, sedimentation effects are very clear and cause the concentration of the larger particles in the lower layer of the flow and the dispersion of the finer particles from the upper region of the flow to the phoenix column. At 600 s and at about 3-4 km from the vent, we can observe a concentration of 200-μm particles at the base of the flow of about $10^{-2.5}$, whereas the concentration of 10-μm particles decreases to about $10^{-3.5}$.

Figures 10 and 11 show the temporal temperature dis-
tributions of the two solid phases inside the pyroclastic flow of eruption A3 and refer to 10 μm and 200 μm, respectively. At 240 s, the two phases are approximately in thermal equilibrium and just very small differences can be observed in the upper layer of the flow. The temperature at the base of the flow can reach 800 °C above the ground atmospheric temperature also at 3.5 km from the vent, whereas a slow decrease can be observed moving toward the most advanced region of the flow. After 420 s from the beginning of the eruption, a thermal nonequilibrium effect appears inside the flow. It is located in the region of the flow with a higher concentration of 200-μm particles, extending from 3 to 4.5 km from the vent. A larger effect can be observed after 600 s, with a higher concentration of 200-μm particles in a wider region of the flow. The gas temperature is not reported in the figure but it is similar to the temperature of particles of 10 μm. No significant difference was

Figure 5. Distribution of the water vapor volumetric fraction in the atmosphere at 30, 120, 420, and 600 s of eruption A2. The contour levels shown are the exponents to the base 10 and, beginning from the outer or distant from the vent region, correspond to -10, -8, -6, -5, -4, -3, -2, -1, and 0.
observed until 600 s between the temperature of the two solid phases neither in the proximal region of the flow (approximately before the phoenix column), nor in the hot particle dilute plumes forming above the fountain and the flow.

The above described nonequilibrium effects inside the flow can be better quantified with Figures 12 and 13 that show sections at constant $z$ of the particle volumetric fraction and temperature of the two solid phases.

Figures 12a and 12b illustrate the behavior of the particle volumetric fraction for the two solid phases. By comparison, at each time, of the curves associated to the two particulate phases, we observe the progressive sedimentation of 200-μm particles which are confined within the first two cells above the ground, whereas the smaller particles maintain a quite uniform concentration. In particular at 600 s, the 200-μm particles have a concentration at the base of the flow of about 1 order
Figure 7. Distribution of solid volumetric fraction of particulate phase 2 (\(d_{\text{p2}} = 200 \mu m\)) at 30, 120, 420, and 600 s of eruption A3. The contour levels shown are the exponents to the base 10 and, beginning from the outer or distant from the vent region, correspond to \(-10\), \(-8\), \(-6\), \(-5\), \(-4\), \(-3\), \(-2\), and \(-1\).

of magnitude larger than that of the smaller particle. Furthermore, from the figure we can note a progressive increase in the concentration of the larger particles in the distal region of the flow, whereas no sedimentation process appears in the proximal region. Figures 13a and 13b illustrate the corresponding values of temperature, at the same elevations of Figure 12, for the two solid phases. As discussed above, the difference between the temperatures of the two solid phases can reach about 300°C as shown from the curves of 10 and 20 m quota, after about 420 and 600 s. The particles in contact with the adiabatic ground wall cool very slowly, and at the ground level the temperature is about 800°C, even after 10 min and at 5.5 km from the vent. The curves for 10, 20, and 50 m height also exhibit a local maximum related to the ascent of the water vapor and particles forming the phoenix column. Finally, one should note the gradual decrease of temperature in the most ad-
Figure 8. Distribution of solid volumetric fraction of particulate phase 1 \((d_{p1} = 10 \, \mu m)\) at 180, 300, and 600 s in the pyroclastic flow of eruption A3. The contour levels shown are the exponents to the base 10 and, beginning from the outer or distant from the vent region, correspond to -10, -8, -6, -5, -4, -3.5, -3, -2.5, and -2.

Discussion

The results obtained from the present numerical simulations strongly highlight the importance of a multiphase formulation of the solid phase of the eruptive mixture. Several works, in the last few years, illustrated the potentiality of two-phase gas-particle flow models in the description of the large-scale behavior of collapsing columns and associated pyroclastic flows. Wohletz et al. [1984], Valentine and Wohletz [1989a], and Dobran et al. [1993] showed that by such description it is possible to explain very complex processes such as the column collapse and the generation of phoenix column. Parametric studies and applications to real eruptions performed by these models stressed the important effect of particle size on the large-scale evolution of the process [Valentine and Wohletz, 1989a; Neri and Dobran, 1994; Giordano and Dobran, 1994; Dobran et al., 1994]. Several nonintuitive phenomena appeared strongly affected by the selected particle size for the solid phase. Among the most relevant is the strongly unstable behavior of the fountain and pyroclastic flow that was observed with particle size of the order of few tens of microns and with particular vent conditions. Such a behavior can cause strong oscillations of the fountain height and of the pyroclastic flow mass flow rate even several kilometers away from the vent [Neri and Dobran, 1994]. Furthermore, particle size produces an evident effect also on the sedimentation process inside the pyroclastic flow and in the generation of the phoenix columns. In particular, particle size of the order of 100-200 \(\mu m\) produce longer travel distance than a 10-\(\mu m\) particle size, larger sedimentation rates, denser and thinner pyroclastic flows, and phoenix columns with very low concentration of particles. However, particle sizes larger than 500-\(\mu m\)
reduce the travel distance of the flow due to the very large sedimentation rate [Giordano and Dobran, 1994; Neri and Dobran, 1994; Dobran et al., 1994].

Simulations A1 and A2, performed by a two-phase flow application of the above presented three-phase model, are consistent with the described general features of collapsing volcanic columns and pyroclastic flow, as resulted from the cited most updated models. Figures 2, 3, 4, and 5 fully justify the above considerations on the density, velocity, sedimentation rate, and phoenix column features as a function of particle size. A comparison between Figures 2 and 4 strongly highlights the different runout of the pyroclastic flow, with a difference of about 2 km between the two simulations, as well as the absence of 200-μm particle size in the ascending phoenix column. Also very different is the thickness of the pyroclastic flow, that for simulation A1 reaches 60-80 m against the 20-40 m of simulation A2. The different velocity propagation of the flow also affects the time of formation of the phoenix cloud that for simulation A1 is more rapid.

The model described in Description of Governing Equations and Numerical Scheme extends the previous models’ formulation considering a system consisting of a binary mixture of particles in addition to a gas phase composed by two chemical components (water vapor and air). The simulation results, illustrated by Figures 6-13, clearly show the importance of including two solid phases, and therefore the present model should be an improvement in simulating the column and pyroclastic flow dynamics. Figures 6 and 7 clearly show the large-scale effects of the binary formulation of the pyroclasts phase. The two particle fraction distributions represented in the figures give evidence of the strong mutual influence between the two particulate phases. Such mutual influence can be already observed in the first minutes of the eruption (see plots at 30 and 120 s of Figures 6 and 7) when the two spatial distributions

Figure 9. Distribution of solid volumetric fraction of particulate phase 2 (dp2 = 200 μm) at 180, 300, and 600 s in the pyroclastic flow of eruption A3. The contour levels shown are the exponents to the base 10 and, beginning from the outer or distant from the vent region, correspond to -10, -8, -6, -5, -4, -3.5, -3, -2.5, and -2.
are very similar. At higher times (see plots at 420 and 600 s) such influence appears still more evident. On the one hand, the strong rising of hot water vapor and 10-μm particles size forming the phoenix column drag upward a small but appreciable quantity of 200-μm particles. In this case a 200-μm particle concentration of about $10^{-8}$ can be noted in a large region of the phoenix cloud, whereas particles of this size were completely absent in the water vapor-rich phoenix column of simulation A2. On the other hand, 200-μm particles force the smaller 10-μm particles to reach runout distances much larger than those reached in simulation A1 with all particles of 10 μm (6.5 km against 5 km after 600 s). Therefore the final result consists of a very different distribution of both solid phases compared to the distributions that were obtained considering just one solid phase. Moreover, the observation of a 200-μm particle concentration of $10^{-5}$-$10^{-8}$ in the hot ascending plumes above the fountain gives evidence of the relevant drag capacity of these convective regions. Although the performed simulations are not representative of a specific eruption, their results are qualitatively consistent with volcanological data obtained from stratigraphic studies. Several coignimbrite columns have been observed in recent as well as past eruptions, such as the eruption of Mount St. Helens in 1980 [Hoblitt, 1986], Tambora (1815) [Sigurdsson and Carey, 1989], Krakatau (1883) [Self and Rampino, 1981], and Toba in Sumatra (75,000 before present) [Rampino and Self, 1992]. Deposit patterns of eruptions characterized by the formation of such large coignimbrite columns generally show that particles of few tens of microns are preferentially separated from the flow and give rise to ascending coignimbrite columns. In these cases very low concentrations of larger particles are observed in the upper layers of the deposit [Cas and Wright, 1987].
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Figure 11. Temperature distribution of particulate phase 2 ($d_{p2} = 200\ \mu m$) at 240, 420, and 600 s in the pyroclastic flow of eruption A3. A contour level in the figure represents the difference between the local gas temperature and the undisturbed atmospheric air temperature at $Z = 0$. The temperature contours, starting from the inner or near the vent region, correspond to 900ø, 800ø, 700ø, 500ø, 300ø, 200ø, 100ø, 50ø, and 0øC.

Figures 8-13 focus on more local aspects of the emplacement mechanisms of pyroclastic flows. In detail, Figures 8, 9, and 12 illustrate the mechanical nonequilibrium between the two particulate phases, whereas Figures 10, 11, and 13 focus on the thermal nonequilibrium between them. From the timewise behavior of the particle volumetric fraction of two solid phases the evolution of the segregation process between them is clearly appreciable. In particular, 10-µm particles are distributed on a wider flow front and tend to separate from the body of the flow. On the contrary, 200-µm particles tend to segregate and deposit on the ground. Maximum values of the larger particle concentration are observed in the region of formation of the phoenix column, whereas smaller particles have a lower and quasi-uniform concentration along the flow. These results appear also consistent with the most common volcanological models of ignimbrites describing the flow as a basal avalanche of a concentrated dispersion of blocks, and lapilli, underlying a more dilute ash cloud of finer particles [Sparks and Wilson, 1976; Walker, 1985; Denlinger, 1987; Cas and Wright, 1987].

The evolution of the temperature distribution for the two solid phases is also particularly interesting. From the plots a thermal equilibrium between the two solid phases (and between the two solid phases and the gas) is evident during the first 5 min of the eruption. At this time, a relevant nonequilibrium effect between the two solid phases appears in the basal region of the flow between 3 and 4.5 km from the vent, where the smaller particles remain in thermal equilibrium with the gas but the larger ones do not. Such a difference between the two temperatures, reaching almost 300°C at 10 m height, can be related to the local increase of the concentration of larger particles, contributing to more effective heat conduction between the 200-µm particles. Therefore our results qualitatively indicate that thick and voluminous flows, characterized by a large amount
of small particles, would have a greater tendency to be nonwelded, whereas dense coarse flows can be densely welded throughout much of their thickness. Such indications are consistent with several studies reported in literature even if emplacement temperature strongly depends on vent conditions and complex mixing processes between the mixture and the atmosphere [Smith, 1960; Sparks et al., 1978].

Present results are qualitatively consistent also with experimental studies aimed at the investigation of the main physical processes that occur during pyroclastic flow emplacement. Carey et al. [1988], Woods and Caulfield [1992], and Woods and Bursik [1994] performed isothermal experiments by the use of fresh water particle laden solutions and/or of chemical solutions showing very similar global features of the system dynamics. In these experiments, high density solutions leaving the vent develop collapsing columns, radially spreading pyroclastic flows, sedimentation of particles along the flow and unstable density gradients which produce rising phoenix columns from the flow. In particular, the deduction of Woods and Bursik [1994] indicating that air entrainment dominated flows are characterized by larger ascending phoenix columns, whereas sedimentation dominated flows produce smaller phoenix columns, appears in agreement with present results (see Figures 2-7).

From the above considerations it is evident that a more realistic modeling of collapsing columns and pyroclastic flows should require the implementation of sev-
eral particulate phases and further investigations of the relevant interphase transport processes taking place between them. The above results have been obtained by only two particulate phases, and therefore it could be dangerous to extrapolate them to real pyroclastic flows. In addition, the employed computational grid does not allow to adequately resolve the strong flow gradients that characterize these phenomena for high sedimentation rates and particle concentrations. In spite of these limits, the employed 10-m grid size along the ground appears to be adequate for the presented simulations of dilute flows and able to represent the large-scale behavior of these phenomena. Furthermore, present results strongly highlight the multiphase nature of these kinds of systems and demonstrate again that thermofluid-dynamics processes can be responsible for very complex and nonintuitive stratigraphic features that can be observed in the field.

Summary and Conclusions

The large-scale physical behavior of collapsing volcanic columns and pyroclastic flows has been investigated by a three-phase thermofluid-dynamics flow model describing pyroclasts by two particulate solid phases. The model accounts for the mixing of hot water vapor and two solid phases leaving the vent with atmospheric air. The transport differential equations describing the conservation of mass, momentum, and energy were nu-
Numerically solved on an axisymmetric physical domain with different percentages of the two particulate phases at the vent.

Simulations with just one particulate phase show very different physical behavior as a function of particle size. Collapsing columns with particle size of the order of 10 μm lead to the formation of thick pyroclastic flows, with short runout distances, and generation of large phoenix columns. Eruptions with 200-μm particle size are characterized by the formation of thin and denser pyroclastic flows, greater runout distances, and by the absence of particle laden phoenix columns. The simultaneous presence of two particulate phases strongly affects the flow mobility and particle concentration and segregation in the various regions of the flow. As a consequence, 200-μm particles are dragged into the phoenix columns by the ascending smaller particles, whereas the latter can now reach longer distances due to the higher mobility of the larger particles. Mechanical and thermal nonequilibrium conditions in the various regions of the flow are also highlighted by the model. Particle segregation effects increase the 200-μm particle concentration in the lower layer of the flow, whereas 10-μm particles tend to leave the flow and rise upward in the phoenix column. Larger particles show also higher temperatures in the basal region of the flow, whereas smaller particles remain in thermal and mechanical equilibrium with the gas phase.

Model applications were limited to the consideration of dilute pyroclastic flows, and therefore of particle sizes of few hundreds of microns, because of the limited resolution of the computational grid. Furthermore, dilute flows permit to neglect viscous dissipation effects that become critical for very dense flow. A higher computational resolution, however, will allow us in the future to account for a wider particles size range and to describe the global column behavior on a smaller scale. The results also appear qualitatively in agreement with previous numerical models, laboratory experiments, and field observations. From this point of view, the verification of model results by ad hoc laboratory experiments and the reconstruction of past eruptions is of paramount importance to improve our confidence in the model. Finally, simulation results strongly highlight the model capability to describe the behaviors of the different components of the eruptive mixture and suggest that future studies are within reach for multiphase and multicomponent modeling of these phenomena.

**Notation**

- $C_{d,k}$: drag coefficient between the gas and the $k$th solid phase.
- $C_P$: specific heat at constant pressure.
- $d$: particle diameter.
- $D_{g,k}$: interfacial drag between the gas and the $k$th solid phase.
- $D_{k,j}$: interfacial drag between the $k$th and $j$th solid phase.
- $D_v$: vent diameter.
- $e$: restitution coefficient.
- $g$: gravitational body force per unit mass.
- $G$: solid elastic modulus.
- $h$: enthalpy.
- $H$: vertical extent of the computational domain.
- $I$: unit tensor.
- $k$: thermal conductivity.
- $L$: radial extent of the computational domain.
- $N_u$: Nusselt number.
- $P$: pressure.
- $Q_k$: volumetric heat transfer rate between the gas and the $k$th solid phase.
- $r$: radial coordinate.
- $R$: gas constant.
- $R_e$: Reynolds number.
- $t$: time.
- $T$: temperature.
- $T$: stress tensor.
- $u$: radial component of velocity.
- $v$: vertical component of velocity.
- $v$: velocity vector.
- $y$: mass fraction of a component in the gas phase.
- $Y$: mass fraction of gas phase in two-phase mixture.
- $z$: vertical coordinate.
- $\alpha$: coefficient for nonhead particle collisions.
- $\beta$: volumetric fraction of gas or solids.
- $\theta$: azimuthal angle.
- $\mu$: viscosity.
- $\rho$: density.
- $\tau$: stress tensor.

**Subscripts**

- $a$: air.
- $c$: collapse, coulombic.
- $g$: gas phase.
- $ge$: gaseffective.
- $gt$: gasturbulent.
- $k$: $k$th solid phase.
- $m$: mean or mixture.
- $s$: solid.
- $v$: vent, viscous.
- $w$: water vapor.

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