A Modified Archie's Law for Two Conducting Phases

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Structure

- Different Mixing Models
- A Modified Archie's Law
- Testing the New Law
- Applications

Mixing Models I

Many Mixing Models have been published

- The Wiener Bounds (Parallel and Perpendicular Models)
- The Hashin-Shtrikman Bounds
- Maxwell's Isolated Sphere Equation and Wagner's Modifications
- The Waff Model
- The Brick-Layer and Modified Brick-Layer Models
- Sphere distribution models
- The Milton-Bergman Models
- Archie's Law

Mixing Models II

Some are valid for:

- One, two, or many conducting phases
- Specified phase geometries (layers, spheres, cracks)
- Complete or limited ranges of volume fractions
- Constrained phase distributions (e.g. diffuse spherical inclusions)

Advantages of Archie's Law

Archie's Law is useful because:

- **It contains a variable parameter that describes the connectivity of a conducting phase**
- **It is valid for complex rock geometries**
- **It is valid for the complete range of volume fractions**

Disadvantages of Archie's Law

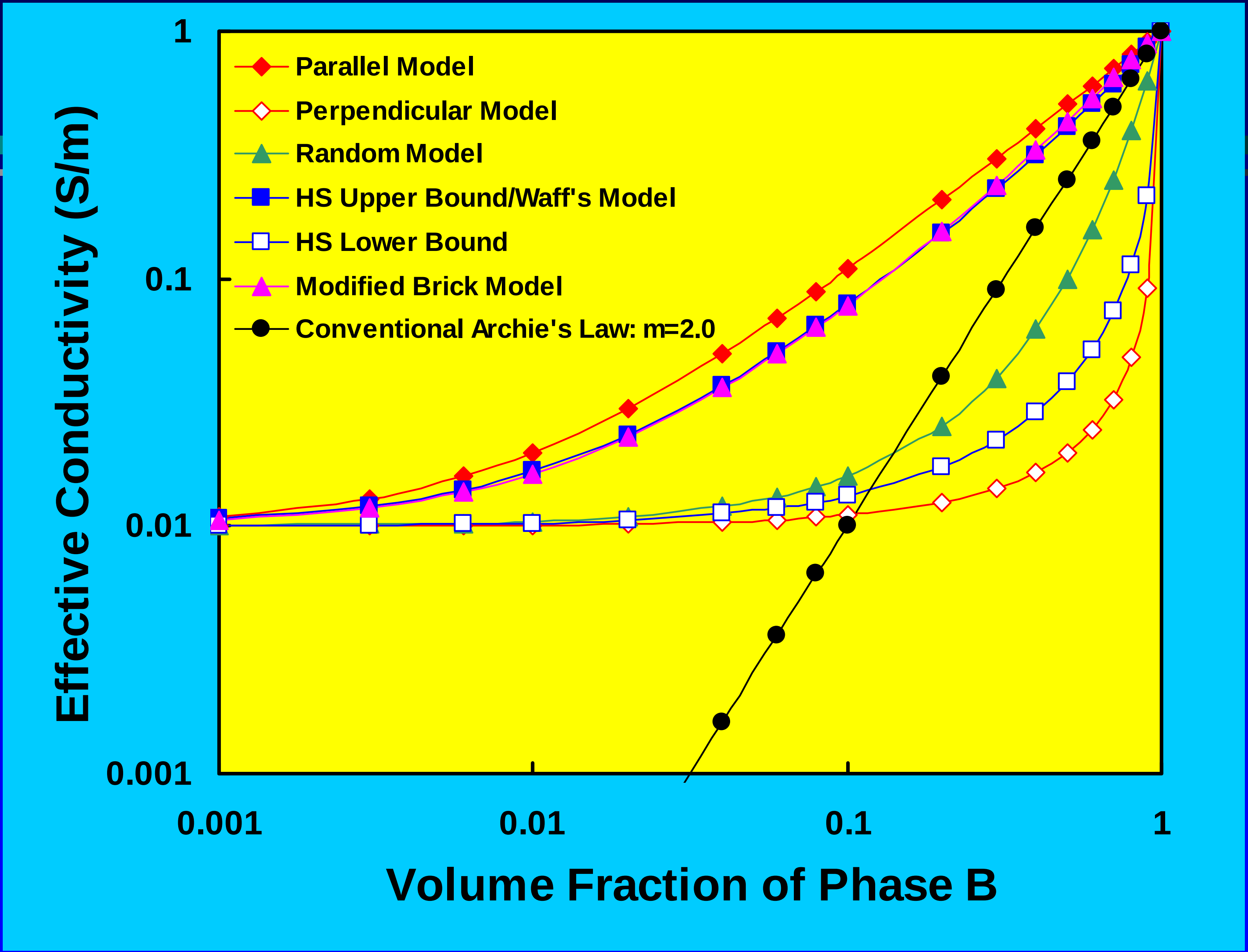
Archie's Law has one BIG disadvantage:

- It is only valid for ONLY ONE conducting phase

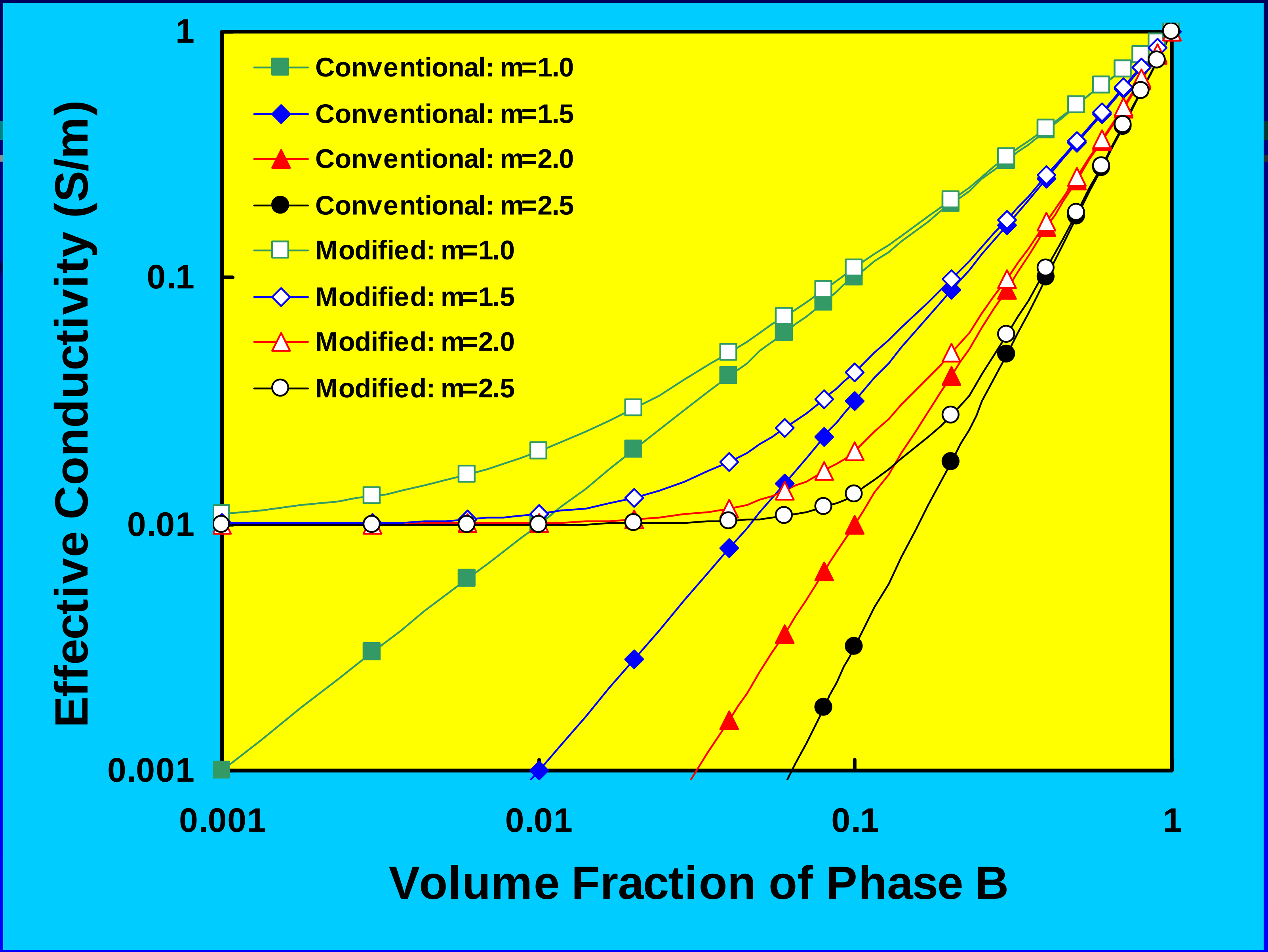
Hence it cannot be used where the rock matrix has a significant conductivity, e.g.

- Partial melting calculations
- Clay-bearing rocks
- Graphite/sulphide bearing rocks
- In the lower crust and mantle

Some Mixing Models



Archie's Law


Conventional: m=1.0
Conventional: m=1.5
Conventional: m=2.0
Conventional: m=2.5
Modified: m=1.0
Modified: m=1.5
Modified: m=2.0
Modified: m=2.5
Effective Conductivity (S/m)
1
0.1
0.01
0.001
0.001
0.01
0.1
1
Volume Fraction of Phase B

A Modified Archie's Law I

- Two Conducting Phases *A* and *B*
- *A* and *B* have volume fractions χ
- where $\chi_A + \chi_B = 1$
- *A* and *B* have electrical conductivities σ_A and σ_B
- The distribution of *A* is described by an exponent *p*
- The distribution of *B* is described by an exponent *m*

Modified Archie's Law - Eq. 1

Neglecting Stiltjes Integral allows us to modify the conventional Archie's Law as:

$$\sigma_{eff} = \sigma_A \chi_A^p + \sigma_B \chi_B^m$$

A Modified Archie's Law - Eq. 2

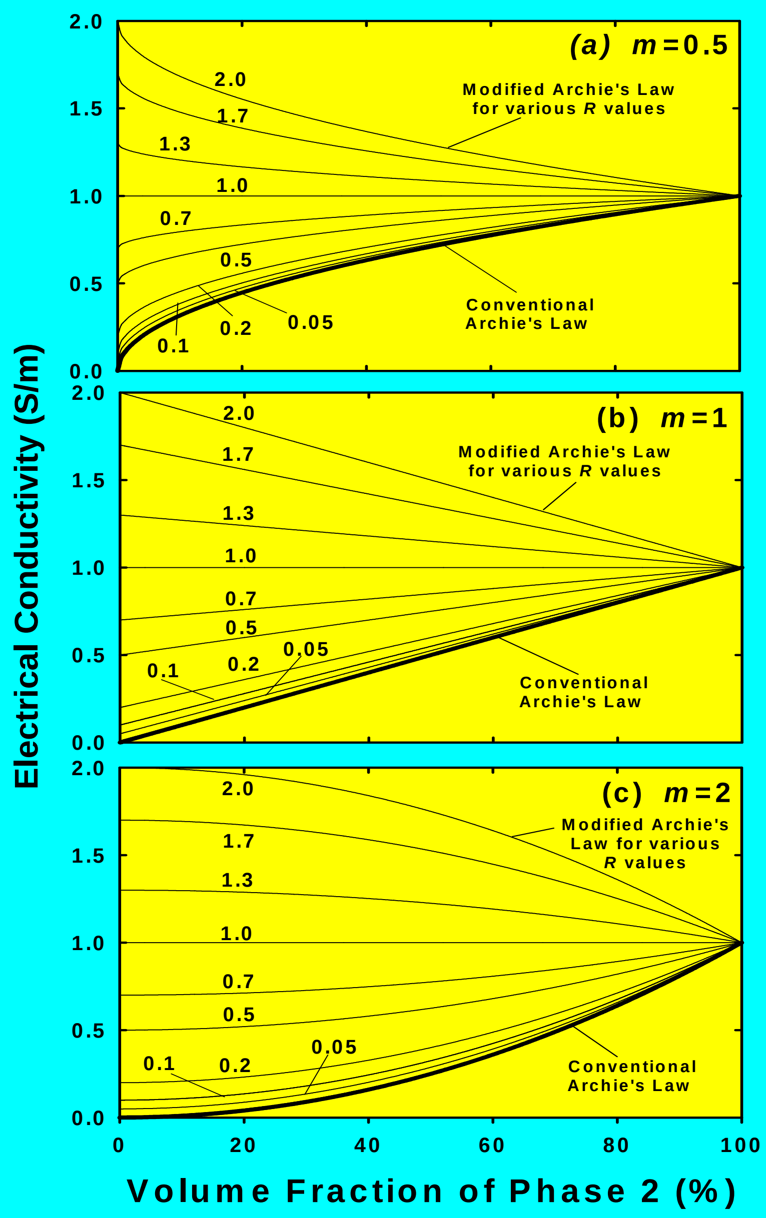
Applying an isoconductivity boundary condition allows us to obtain the exponent *p* as a function of the exponent *m* (or *vice versa*):

$$p = \frac{\log\left(1-\chi_B^m\right)}{\log\left(1-\chi_B\right)}$$

A Modified Archie's Law - Final

Hence

$$\sigma_{eff} = \sigma_A (1-\chi_B)^{(\log(1-\chi_B^m)/\log(1-\chi_B))} + \sigma_B \chi_B^m$$



(a) m=0.5
Modified Archie's Law for various R values
Conventional Archie's Law
2.0
1.7
1.3
1.0
0.7
0.5
0.2
0.05
0.1
(b) m=1
Modified Archie's Law for various R values
Conventional Archie's Law
(c) m=2
Modified Archie's Law for various R values
Conventional Archie's Law
Electrical Conductivity (S/m)
Volume Fraction of Phase 2 (%)
0
20
40
60
80
100

Testing The Modified Law

- 10 samples made from conductive Cu_2O grains
- Different grain sizes and hydrostatic compaction
  → different porosities
- Saturated with different brine salinities/conductivities
- Complex electrical conductivity measured at 50 frequencies (50 Hz to 500 kHz) with 2 black-Pt electrodes
- Porosity measured by He method before and after
- Brine conductivity measured under same conditions

Validation of the Modified Law

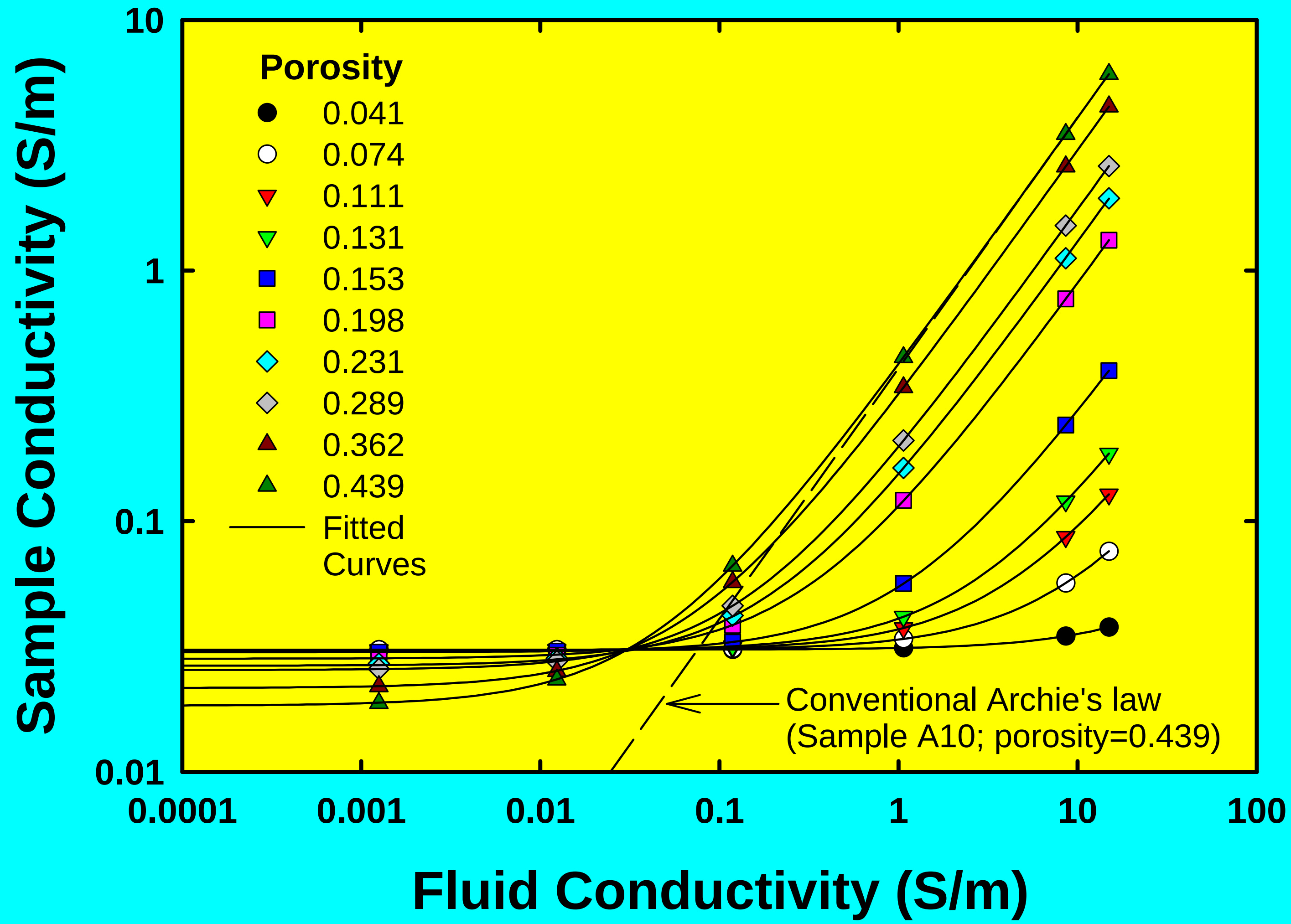
- Modified equation fitted to σ_{eff} versus σ_{brine} data
- Coefficient of determination R^2 >0.99 for all samples
- Hence 1st equation is correct

- Crossplot of measured *p* against *p* predicted from *m* and measured *m* against *m* predicted from *p*
- 1:1 relationship
- Hence 2nd equation is correct

- Hence modified form of Archie's Law is validated

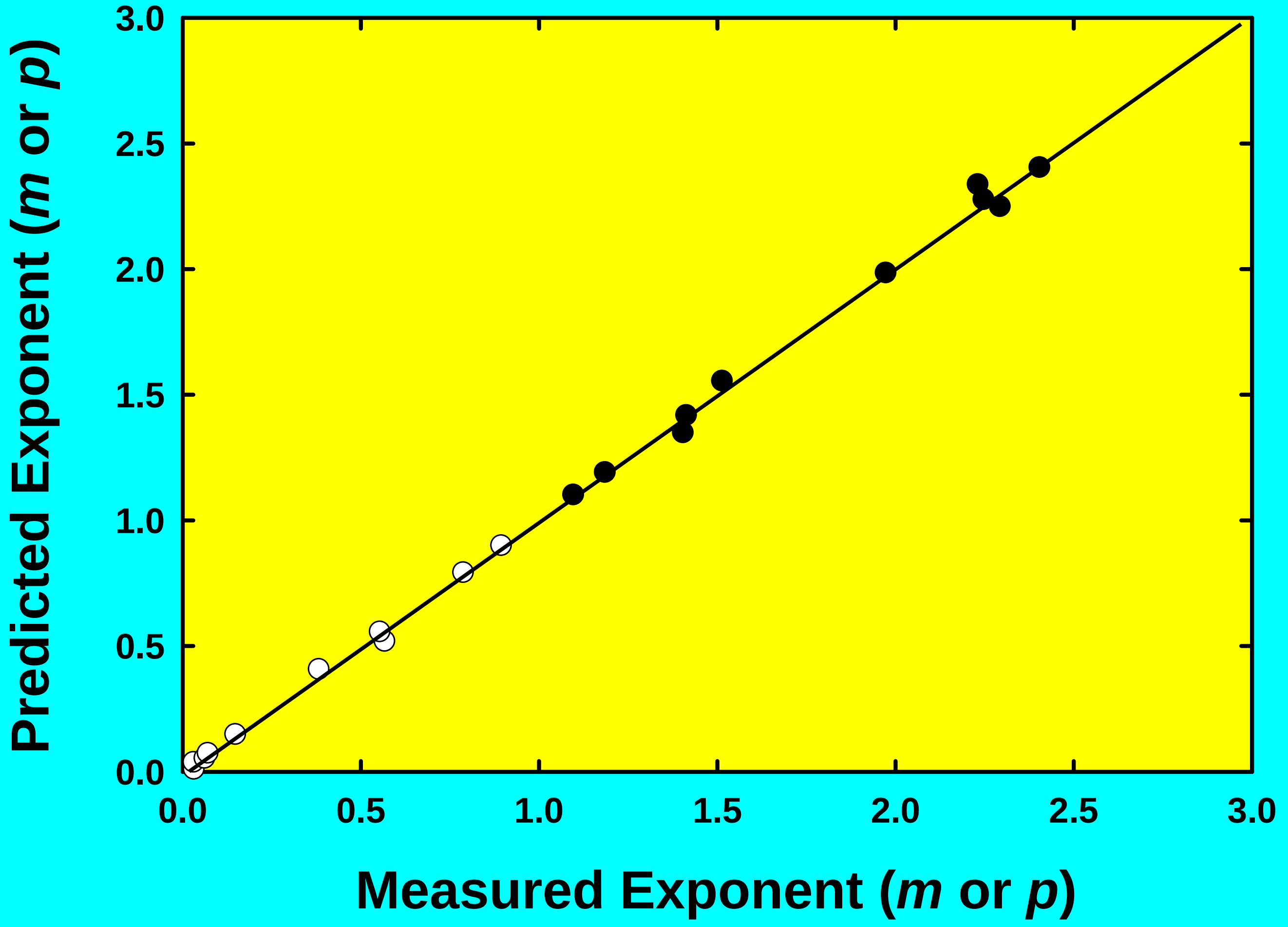
Test of Eq. 1

R^2>0.99



Test of Eq. 2

1:1 Straight Line indicates Eq. 2 is correct



Application to Partial Melt - Effective Conductivity Calculations

IF:	**Matrix conductivity**	σ_A **= 0.015 S/m**
	Melt conductivity	σ_B **= 0.3 S/m**
	Partial melt fraction	χ_B **= 10%**

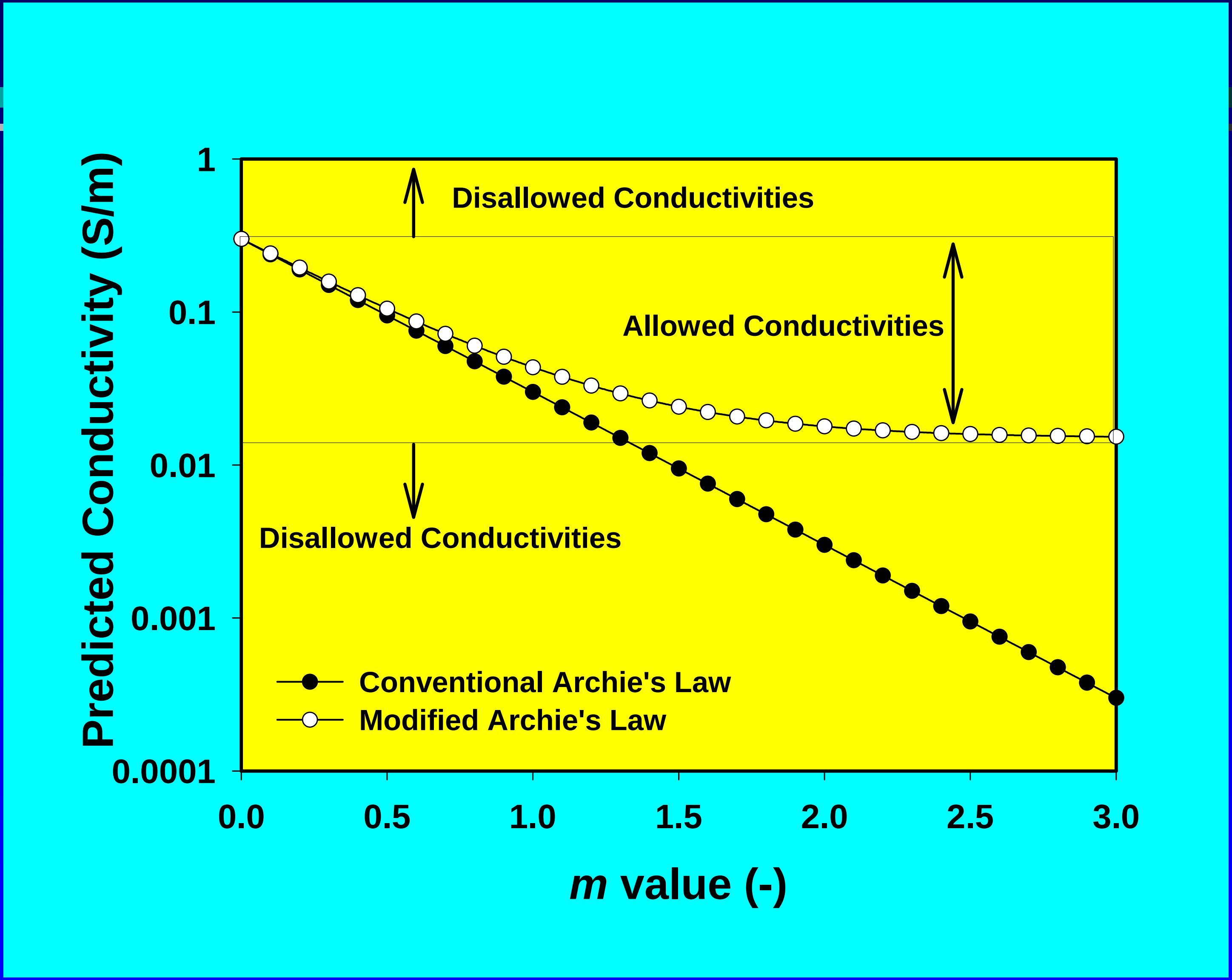
Conventional Archie's Law gives σ_{eff} **= 0.003 S/m with** $m=2$

Modified Archie's Law gives σ_{eff} **= 0.0178 S/m with** $m=2$

- **Conventional law in error as** $\sigma_{eff} < \sigma_A$ *or* σ_B
- **Modified law gives** $\sigma_A < \sigma_{eff} < \sigma_B$ **as expected**

Application to Partial Melt

Effective Conductivity Calculations



Application to Partial Melt - Partial Melt Fraction Calculations

IF:	Matrix conductivity	σ_A= 0.015 S/m
	Melt conductivity	σ_B= 0.3 S/m
	Effective conductivity	σ_{eff}= 0.0178 S/m

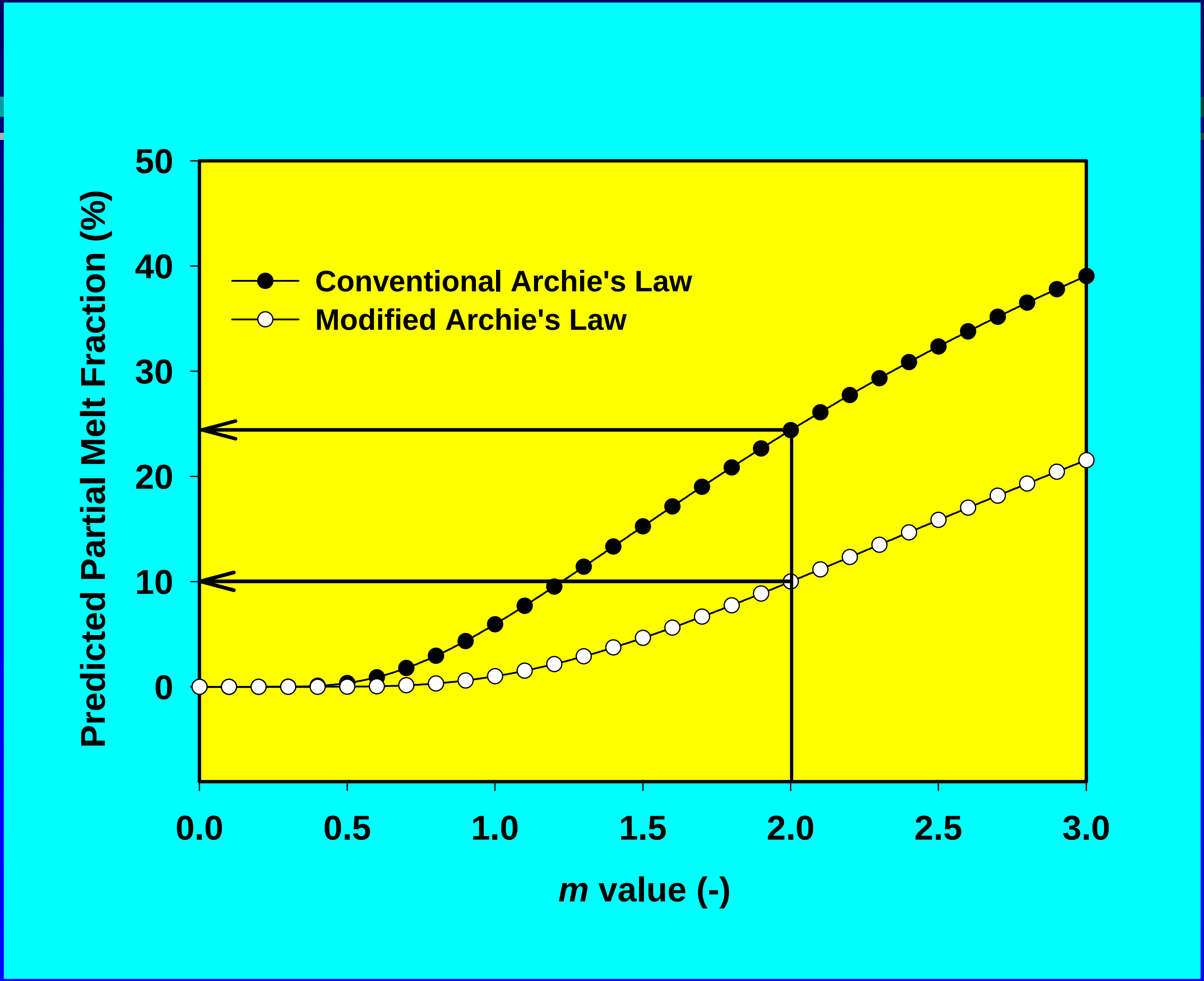
Conventional Archie's Law gives χ_B = 24.4% for m=2

Modified Archie's Law gives χ_B = 10% for m=2

- Large difference between results from the conventional law and from the modified law
- Because the conventional law neglects the significant conductivity of the matrix

Application to Partial Melt

Partial Melt Fraction Calculations



Summary

- Archie's law can only be used with ONE conducting phase
- We have modified Archie's law to enable it to be used with TWO conducting phases
- The modified equation is relatively simple to apply, and retains parameters that describe the connectivity of each conducting phase
- The modified equation has been validated and been shown to work well
- The modified law can be used to improve the assessment of partial melting at depth

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