How are grain size and pore size related? A transformation based on electro-kinetic theory

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INTRODUCTION

Most permeability models use effective grain size or effective pore size as an input parameter. Until now, an efficacious way of converting between the two has not been available. We propose a simple conversion method for effective grain diameter and effective pore radius using a relationship derived by comparing two independent equations for permeability, based on the electro-kinetic properties of porous media. The relationship, which we call the theta function, is not dependent upon a particular geometry and implicitly allows for the widely varying style of microstructures exhibited by porous media by using porosity, cementation exponent, formation factor, and a packing constant. The method is validated using 22 glass bead packs, for which the effective grain diameter is known accurately, and a set of 188 samples from a sand-shale sequence in the North Sea. This validation uses measurements of effective grain size from image analysis, pore size from mercury injection capillary pressure (MICP) measurements, and effective pore radius calculated from permeability experiments, all of which are independent. Validation tests agree that the technique accurately converts an effective grain diameter into an effective pore radius. Furthermore, for the clastic data set, there exists a power law relationship in porosity between effective grain size and effective pore size. The theta function also can be used to predict the fluid permeability of a sample, based on effective pore radius. The result is extremely good predictions over seven orders of magnitude.

MICROSTRUCTURAL PARAMETERS

In this thought experiment (which is described in more detailed in Glover & Walker, Geophysics, 2008) we examine whether there is a reduction in a number of properties (grain size, porosity, connectivity, grain shape) leads to a reduction in the pore size (while keeping the other parameters constant).

For grain size: if the grain size is reduced but the porosity remains constant, the pore size must also diminish.

For porosity: if the porosity reduces with the same grain size, the pore size must diminish.

In the case of connectivity, we alter the connectivity by moving the grains that are marked X. Two scenarios are considered. The first results in a loss of connectivity and pore size only in the horizontal direction (cf. A and B). The second results in a loss of connectivity and pore size in both directions (cf. C and D).

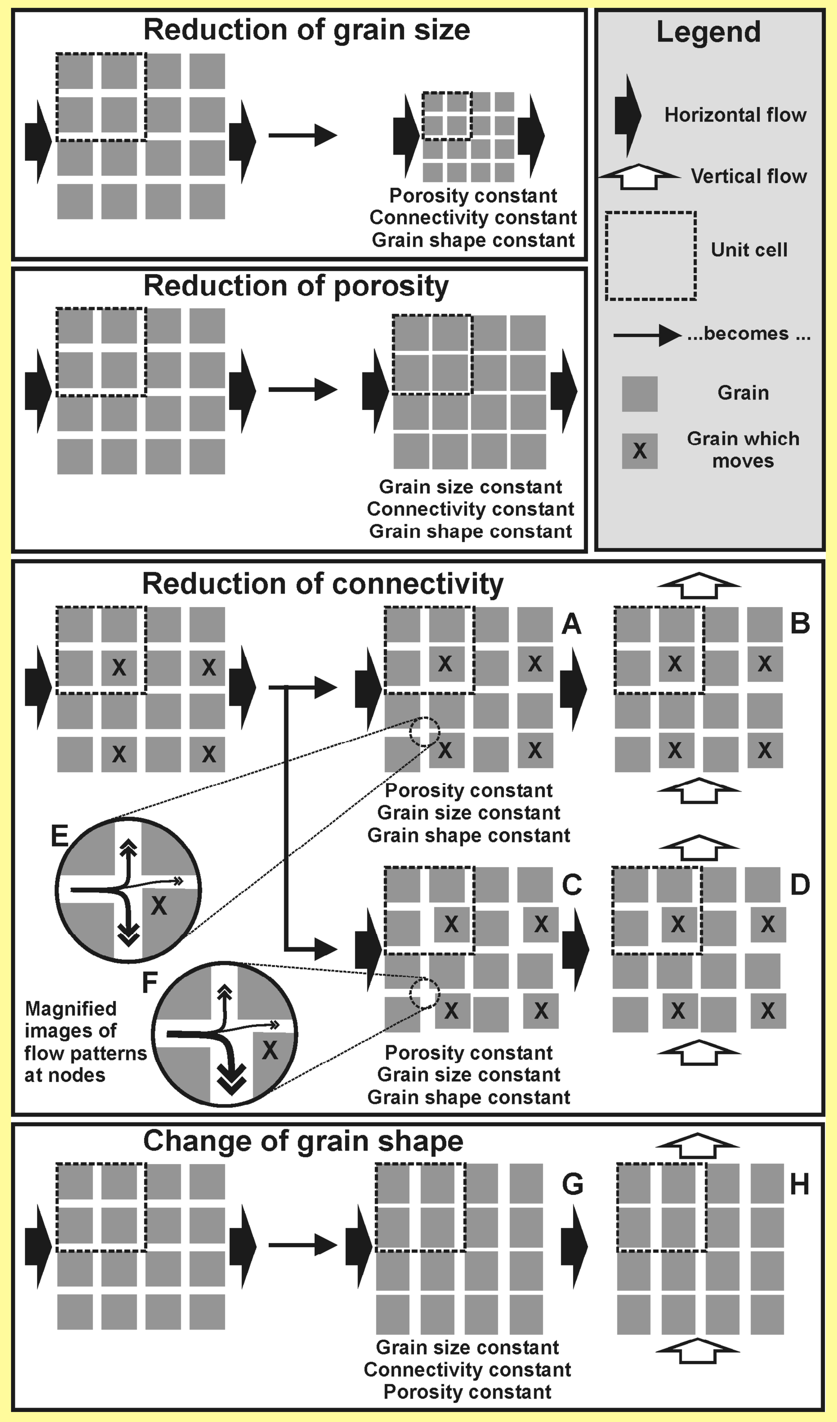
For grain shape: if the grains are elongated while keeping the same mean size, the connectivity and pore sixe in one direction is reduced while it is augmented in the other direction.

In summary

(1) The mean physical pore width of a 2D porous medium is a function of grain size, porosity, and grain shape.

(2) The mean effective pore width of a 2D porous medium is a function of grain size, porosity, connectivity, and grain shape.

(3) Changes in connectivity and grain shape can impose anisotropy in the porous Network that result in changes to the mean physical and mean effective pore widths.



We hypothesize that these relationships are also valid for 3D porous media that consist of a range of grain sizes and shapes.

THE GRAIN SIZE/PORE SIZE RELATIONSHIP

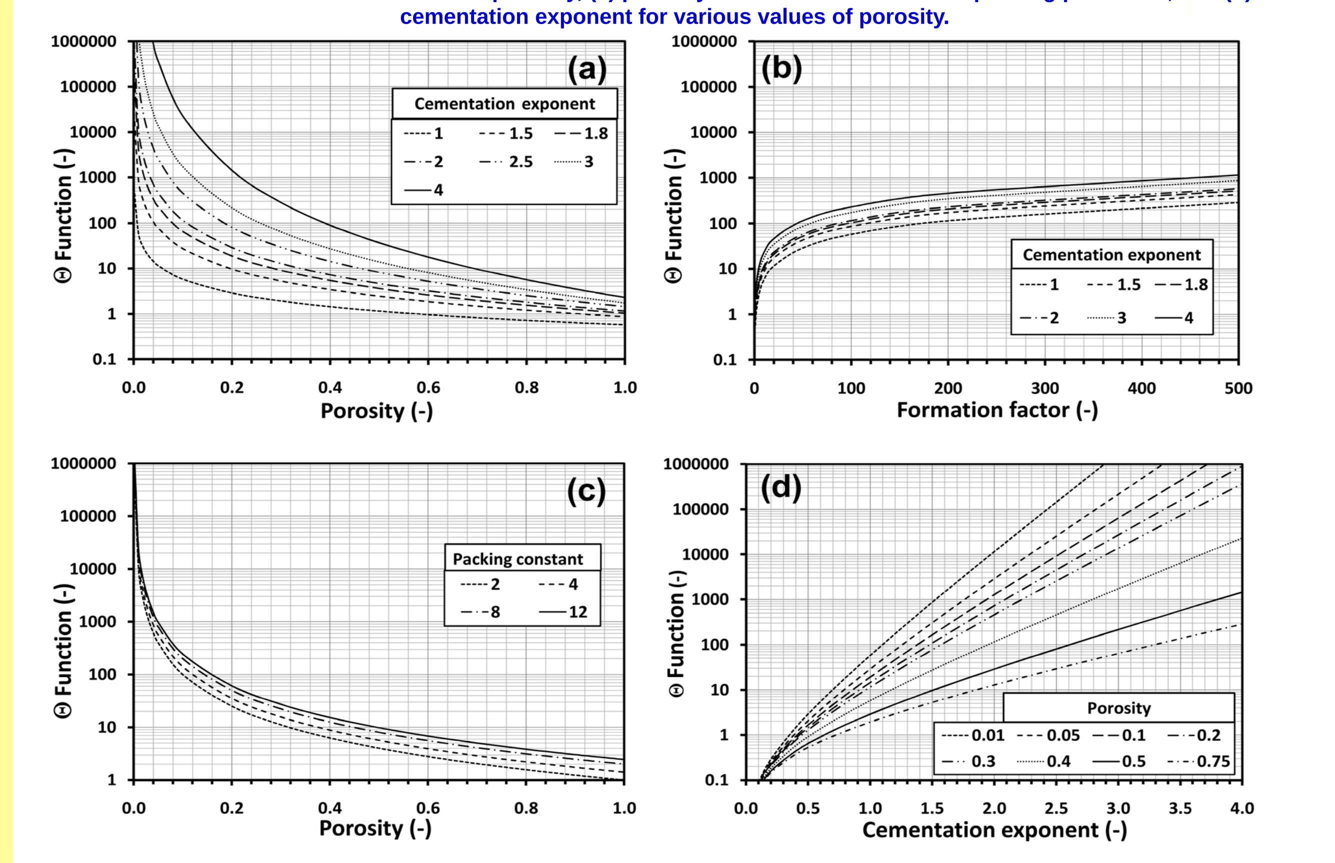
The new effective grain size to effective pore size transformation is derived analytically from considerations of the electro-kinetic coupling between fluid flow and electrical flow in a porous medium. It is not linked to any geometrical considerations that would restrict its use to a given simplified geometry, for example sphere packs. The derivation of the RGPZ model is given in Glover, P.W.J. & Walker, E., A grain size to effective pore size transformation derived from an electro-kinetic theory, Geophysics, 74(1), E17-E29, 2009.

The new transformation the effective grain diameter and effective pore radius is given by

$$d_{eff} = 2\Theta\, r_{eff} \qquad \Theta = \sqrt{\frac{am^2}{8\phi^{2m}}} = \sqrt{\frac{am^2F^2}{8}}$$

where Θ is the transform (unitless), d_{eff} is the effective grain diameter (in meters), r_{eff} is the effective pore radius (in meters), ϕ is the porosity, m is the cementation exponent, F is the formation factor, and a is a parameter that is thought to be equal to 8/3 for three dimensional samples composed of quasi-spherical grains. Figure 2 shows the behaviour of Θ as a function of its major parameters over ranges exceeding those commonly encountered in reservoir rocks.

Figure 2. The theta transformation as a function of (a) porosity for various values of cementation exponent, (b) formation factor for various values of porosity, (c) porosity for various values of the packing parameter, and (d) cementation exponent for various values of porosity.



EXTENSION TO PERMEABILITY

Combination of the theta transformation with the equations of Glover et al. (2006) and Schwartz et al. (1989).

$$\Lambda = \frac{d}{2mF} \qquad k \approx \frac{\Lambda^2}{aF}$$

We can obtain

$$k \approx \frac{\Lambda^2}{aF} = \frac{r_{eff}^2}{8F}$$

which is consistent with the results of Johnson et al. (1986) and Avellaneda and Torquato (1991).

The permeability hence becomes

$$k \approx \frac{r_{eff}^2\chi\phi}{8}$$

and for spheres

$$\Lambda = \frac{r_{eff}}{\sqrt{3}} \qquad k \approx \frac{r_{eff}^2\phi^{3/2}}{8}$$

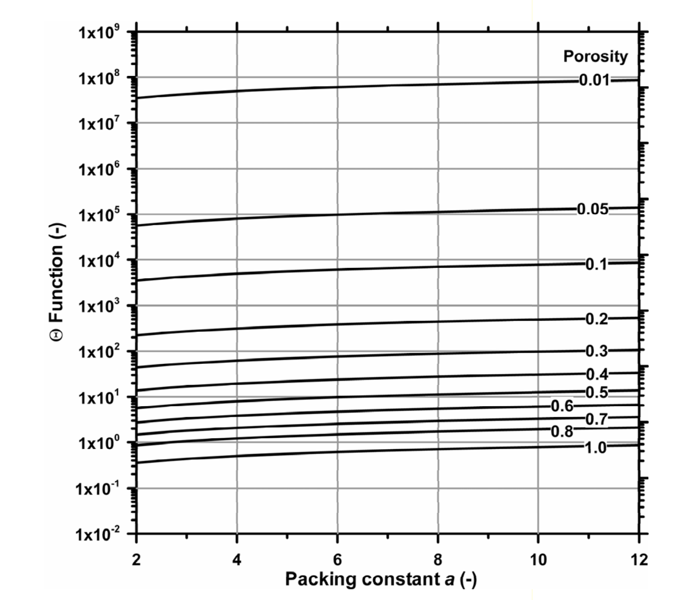
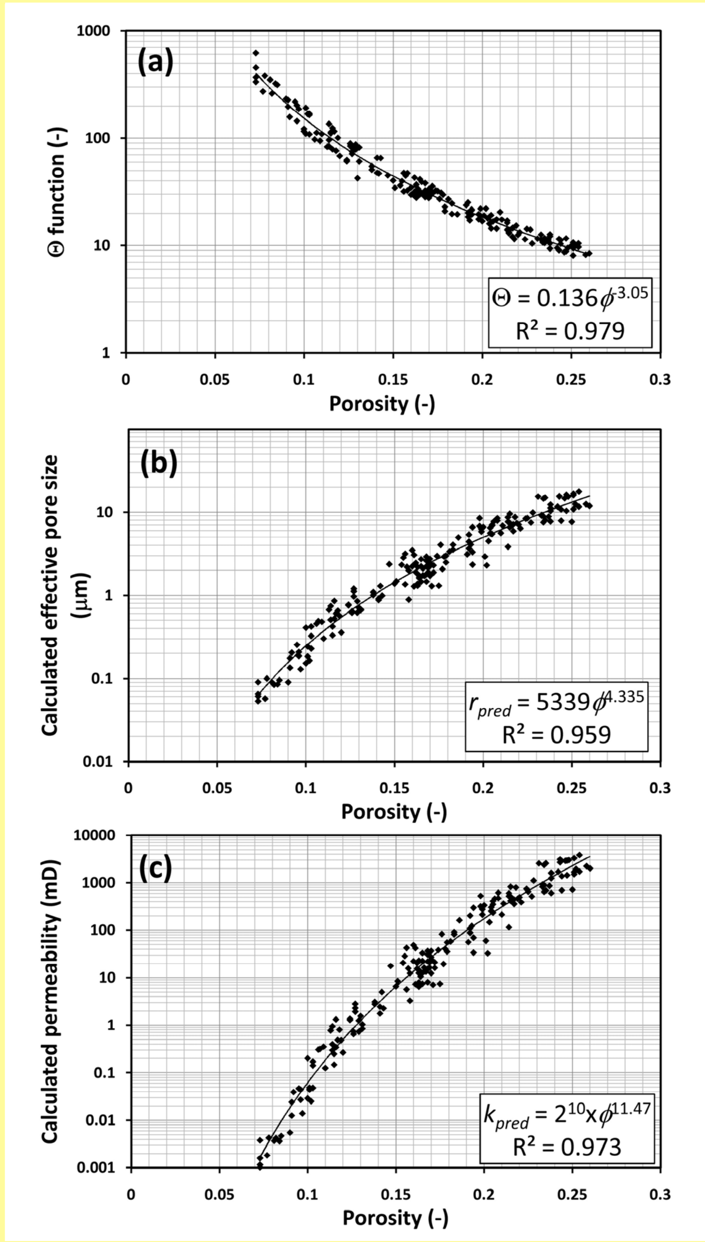


Figure 3. The theta transformation as a function of the packing parameter for various values of porosity.

It can be seen that Θ is relatively insensitive to changes in this parameter; Θ changes by a factor of approximately 3 over the range of packing constants between 2 and 12 (Fig. 3), whereas it changes by over 8 orders of magnitude as a function of porosity and by about 3 orders of magnitude as a function of the cementation exponent at a porosity of 0.2 (Figure 2a above). If the theta function is used subsequently to predict permeability, the permeability is independent of the packing constant.

COMPARISON WITH RESERVOIR DATA

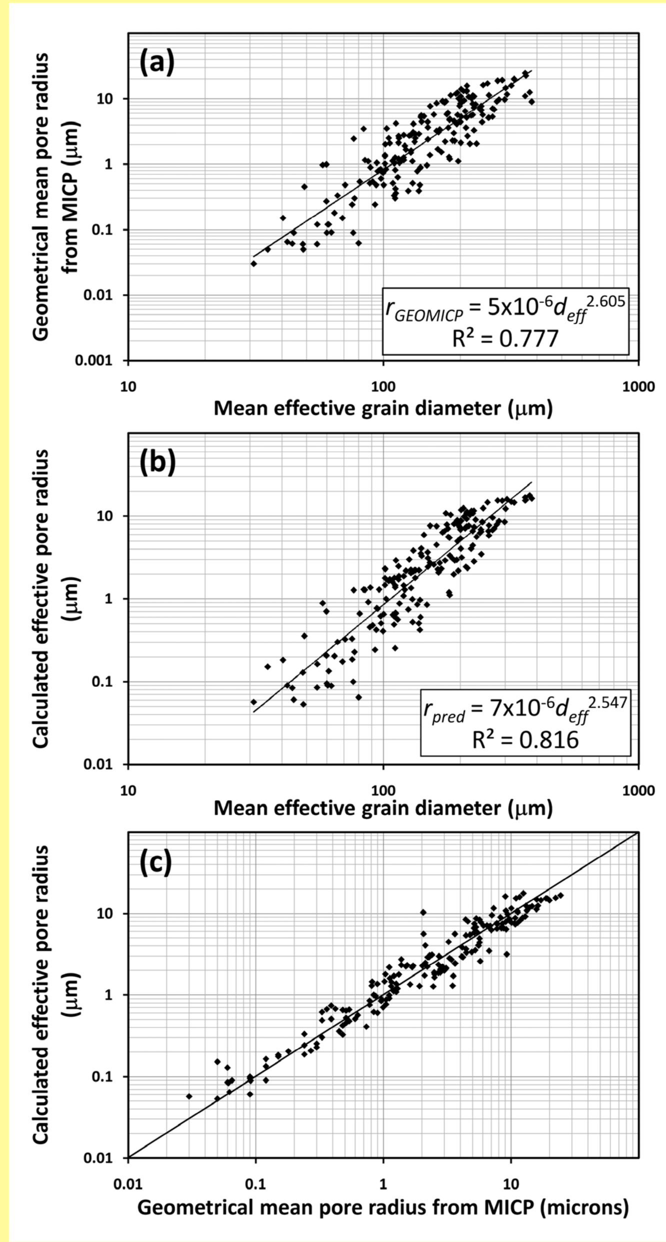


Prediction of grain size for the North Sea data set 188 cores. The original data contained formation factor, porosity and effective grain diameter.

(a) The calculated theta function as a function of porosity, *a*=8/3.

(b) The effective pore radius calculated from the theta function and the mean effective grain diameter as a function of porosity, *a*=8/3.

(c) The fluid permeability predicted from the calculated effective pore radius and the formation factor.

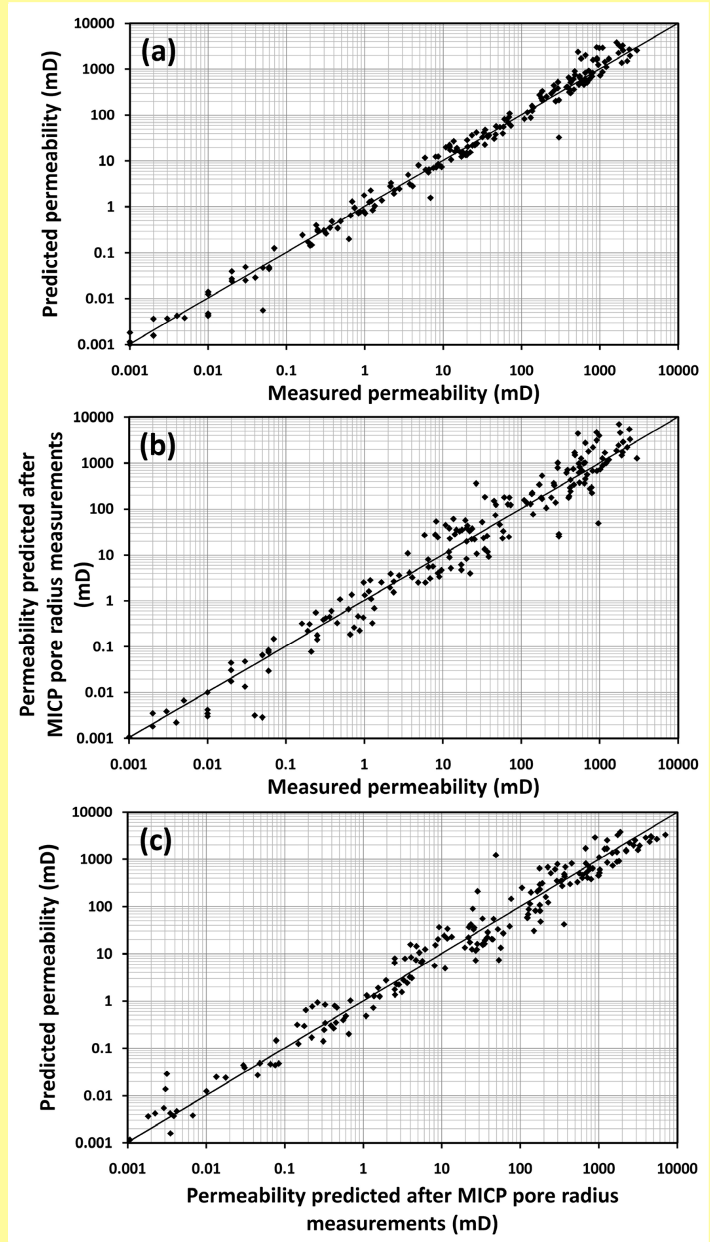


The relationships between measured and predicted effective pore radii and the effective grain diameter for the North Sea data set 188 cores.

(a) The geometric mean effective pore radius from MICP measurements as a function of the measured mean effective grain diameter.

(b) The calculated effective pore radius calculated from the theta function and the effective grain diameter as a function of the measured mean effective grain diameter.

(c) The effective pore radius calculated from the theta function and the effective grain diameter as a function of the geometric mean effective pore radius from MICP measurements.



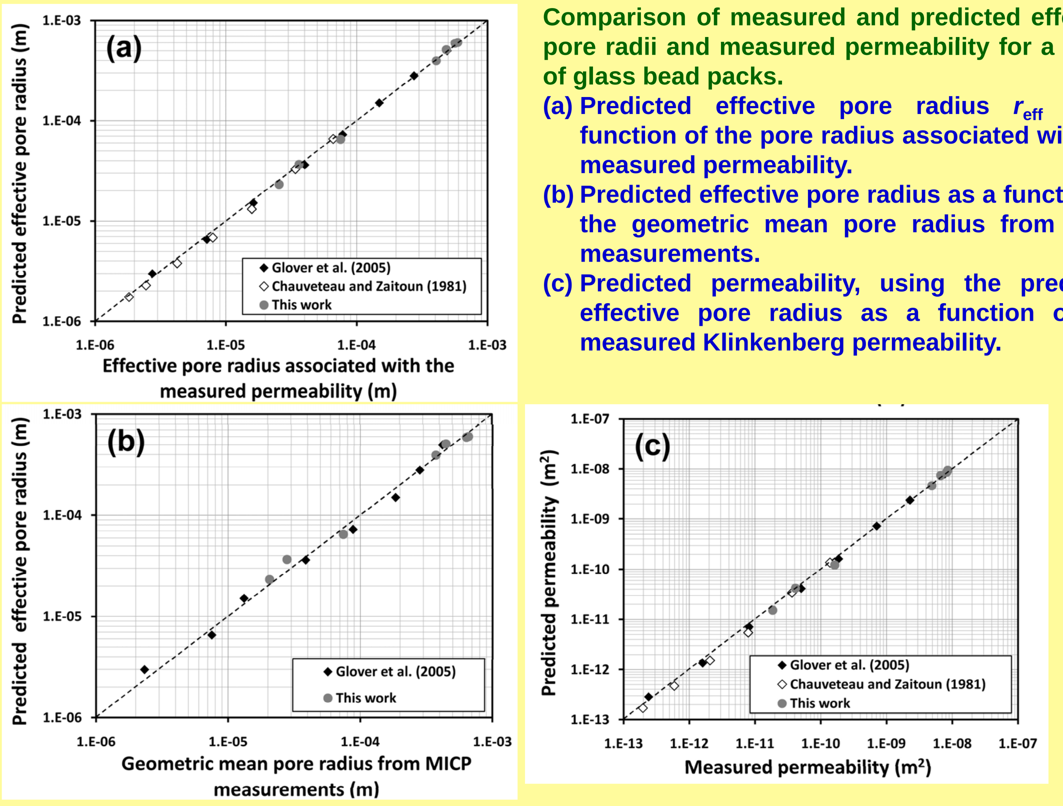
The relationships between measured permeability, permeability calculated using the predicted effective pore radii, and permeability predicted using the geometric mean effective pore radius from MICP measurements for the North Sea data set 188 cores.

(a) The permeability predicted using the effective pore radii as a function of the measured permeability.

(b) The geometric mean effective pore radius from MICP measurements as a function of the measured permeability.

(c) The permeability predicted using the effective pore radii as a function of the permeability predicted using the geometric mean effective pore radius from MICP measurements.

COMPARISON WITH GLASS BEAD DATA



Comparison of measured and predicted effective pore radii and measured permeability for a range of glass bead packs.

(a) Predicted effective pore radius r_{eff} as a function of the pore radius associated with the measured permeability.

(b) Predicted effective pore radius as a function of the geometric mean pore radius from MICP measurements.

(c) Predicted permeability, using the predicted effective pore radius as a function of the measured Klinkenberg permeability.

CONCLUSIONS

1. We have proposed a function for the transformation of effective grain diameter to effective pore radius in all porous media. The function has been derived by comparing equations representing the permeability of porous media that arise from electro-kinetic theory.
2. The function has been validated using glass bead data 22 different diameters found in several published papers, supplemented by tests conducted in our laboratory as well as data from a suite of 188 clastic sandstone core plugs.
3. The validation compared (1) the predicted effective grain size with independent measurements of grain size by image analysis measurements and (2) the predicted effective pore size with the independent measurement of pore access radius from MICP measurements.
4. In all cases, the theta function describes the transformation between effective grain size and effective pore radius very well..
5. We validated our method with glass beads and clastic sandstones. Although there is no fundamental reason why our method should not be valid for carbonate rocks, it has not been tested on these types of rocks.
6. The method is not valid for tight fractured rocks because of the limitation that $F>1$.
7. All of the samples used to validate the model had well-developed unimodal grain-size distributions. The derivation of the transformation is not based on geometric considerations, so the method should apply equally well to rocks with bimodal or more complex grain- and pore-size distributions and even for anisotopic rocks.
8. We also found that the effective pore radius calculated by the transformation is ideal for predicting permeability using relationships proposed by Johnson et al. and Avellaneda and Torquato and, by definition, by using the relationship proposed by Glover et al. The method has been used to predict the permeability of a range of sedimentary rocks for which values of porosity, grain size, and cementation exponent were available from previous publications. Comparison with the relevant measured permeabilities shows that this approach to predicting permeability of reservoir rocks is highly accurate.

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