A differential interpretation of the cementation exponent

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Oil

Discoveries in 2003	182,000,000,000 bbl.	4,500,000,000,000 US$

Johnson et al., 2004

Oil & Gas

Between 1950 and 2002	1,500,000,000,000 bbl. oil	7.5 Tscf gas

Bentley, 2002

Over half of these resources has already been produced, and has driven the global economy for the last fifty years.

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Discoveries made using some expertise in

- Geology, Geophysics, Engineering, and
- Plain Good Luck

However

ALL reserves calculations were made using petrophysics measurements and Archie`s equations

It is difficult to overestimate the impact of either the petrophysical techniques or Archie`s relationships on the worldwide economy.

What is the cementation exponent?

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Archie`s laws link the electrical resistivity to porosity, the resistivity of the pore water, and to the fractional saturation of the pore space with the water.

Used to calculate the hydrocarbon saturation of the reservoir rock hence the hydrocarbon reserves.

Contain two exponents, m and n, which Archie called the cementation exponent and the saturation exponent, respectively.

The conductivity of the hydrocarbon saturated rock is highly sensitive to changes in either exponent.

What is the cementation exponent?

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The cementation exponent commonly takes values from just over 1 to around 5.

Water and oil saturations calculated with Archie's equations are highly sensitive to this level of variability in the cementation exponent.

Thankfully, there are a number of ways in which the cementation exponent can be calculated with precision,

which is why it has often been relegated to the status of a fitting parameter and why no one has tried to understand its physical meaning.

Traditional interpretations

$$F = \frac{\rho_o}{\rho_w}$$

Resistivity formation factor F

$$F = \phi^{-m}$$

Archie's first law

$$m = -\frac{\log(F)}{\log(\phi)}$$

Practical definition

$$\rho_o = a \rho_w \phi^{-m}$$

False 'a'

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Interpretations of m

(1) A factor related to the cementation of the rock (Archie, 1942).

(2) Something to do with the degree of connection of the pores.

(3) A fitting exponent in an empirical relationship.

(4) Only analytically defined for tubes (m=1) and spheres (m=1.5).

(5) The power of a fully analytical equation (Ewing and Hunt, 2006).

(6) Minus the gradient of F/ϕ relationship in log-log space.

Conductivity regime

Need to use conductivity in place of resistivity

Not trivial – but fundamental

We use resistivity for purely traditional reasons (Schlumberger bros., 1927)

However, conductivity has better physics pedigree

$\boxed{\mathbf{J} = \sigma \mathbf{E} = -\sigma\, grad\, V}$ where $\boxed{\sigma = n \beta q}$

Connectedness I

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Now define a conductivity formation factor, G

$$G \equiv \frac{\sigma_o}{\sigma_w} = \frac{1}{F}$$

G, like F, is also approximately constant for a given facies.

The conductivity formation factor varies from zero, which represents the case where $\sigma_o = 0$ (i.e., when $\phi \to 0$) and increases as the porosity increases, with $G \to 1$ (i.e., $\sigma_o = \sigma_w$) as $\phi \to 1$.

G is the conductivity of the rock normalised to the conductivity of the saturating fluid.

G describes the conductivity of a solid/fluid mixture relative to a sample composed only of the fluid.

Connectedness II

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G is a dilution factor where the pore fluid is diluted by rock grains.

G is a dilution factor where the conductivity of the rock is not only affected by the replacement of a given volume of fluid with the same volume of solid matrix, but also by the arrangement of the resulting solid matrix.

Hence, *G* is also a measure of the availability of pathways for electrical transport.

G is, infact, a measure of connectedness of the pore and fracture network of a sample.

Hence we will define *G* to be the *connectedness* of a porous medium.

Connectivity I

A restatement of Archie's first law in the conductivity regime uses the following relationships

$$G = \phi^m \qquad m = \frac{\log(G)}{\log(\phi)} \qquad \sigma_o = \sigma_w \phi^m$$

No better physical interpretation of m than their equivalents in the resistivity regime.

If we define a connectivity $\chi \equiv \frac{1}{\tau}$ hence $\chi = \phi^{m-1}$

Connectivity II

Hence the connectedness becomes

$$G = \phi^{m} = \phi\,\phi^{m-1} = \phi\chi$$

The connectedness G of a rock is due to

(1)The amount of pore volume available for electrical conduction (porosity ϕ), and

(2)The way that that porosity is arranged in three dimensions (represented by the connectivity χ)

Differential form

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The rate of change of connectedness with porosity is

$$\frac{dG}{d\phi} = m\phi^{m-1} = m\chi$$

The rate of change of connectedness with porosity and connectivity is

$$m = \frac{d^2G}{d\chi\, d\phi}$$

Cementation exponent – Physical meaning

The connectedness describes how the conductivity of 100% fluid is modified by the presence of solid non-conducting grains.

The cementation exponent is the sensitivity of the connectedness to changes of connectivity and porosity.

$$m = \frac{d^2 G}{d\chi \, d\phi}$$

Connectivity/porosity relationship I

Differentiating $\frac{dG}{d\phi} = \frac{d(\chi\phi)}{d\phi} = m\chi$ as a product

gives $\frac{d\chi}{d\phi} = \frac{\chi(m-1)}{\phi}$

The rate of change of connectivity of a rock with porosity depends upon

(1) its initial connectivity,
(2) the cementation exponent, and
(3) the initial porosity

Connectivity/porosity relationship II

A large initial connectivity will augment the change in connectivity	A large initial porosity has the effect of diminishing the change in connectivity
If you add a link between pores or cracks in a well connected pore network the result is that the network increases its connectivity more than if the same link were added to a low connectivity network.	If you add a crack to an otherwise low porosity rock the connectivity will change more abruptly than adding the same crack to a rock that already has a high porosity.

Conclusions I

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Connectedness G of a porous medium is defined as the availability of pathways for transport.

Connectedness G is the inverse of the formation resistivity factor, or the conductivity formation factor.

Connectivity χ is the measure of how the pore space is arranged.

Connectivity χ is given by $\boxed{\chi = \phi^{m-1}}$ and depends upon the porosity and the cementation exponent m.

Connectedness G $\boxed{G = \phi\chi}$ depends upon the amount of pore space (porosity ϕ) and the arrangement of the pore space (connectivity χ).

Conclusions II

The rate of change of connectedness with porosity depends upon the connectivity χ and the cementation exponent m.

$$\frac{dG}{d\phi} = m\chi$$

The rate of change of the connectedness with porosity and connectivity is equal to the cementation exponent,

$$\frac{d}{d\chi}\left(\frac{dG}{d\phi}\right) = m$$

Hence, the cementation exponent is interpreted as being the rate of change of the connectedness with porosity and connectivity.

$$m = \frac{d^2G}{d\chi\, d\phi}$$

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