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Modelling the frequency dependence of hydraulic flow and streaming potential coupling coefficients in capillary bundles and porous rocks

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INTRODUCTION

The greatest understanding of the subsurface comes from the comparison of data from diverse sources or different physical properties. Occasionally apparently separate physical properties are physically coupled, and when that happens it is possible for us to predict one property from the other. The electro-kinetic and seismo-electro-kinetic phenomena are examples of such couplings that link the passage of seismo-acoustic vibrations, fluid flow and electrical flow in reservoir rocks.

Electro-kinetic phenomena. A fluid flowing through a reservoir rock moves ions in such a way that an electrical potential difference is created, and an electrical current flows to restore the balance, or vice versa. If one knows the physics behind the coupling, one might in principle calculate the permeability of a rock from an electrical measurement without recourse to empirical data fitting. What is more, since electrical parameters may be measured remotely by self-potential, (magneto-)telluric and GPR techniques, these data may be interpreted as being caused by regional or local fluid flow in a reservoir or other permeable system. Already self-potential measurements have been used to map the convective flow of fluids around volcanoes using electro-kinetic coupling. The same approach can be used to monitor the depletion of a water-driven reservoir, or the flow of water into a potentially seismically active fault prior to an earthquake.

Seismo-electro-kinetic phenomena. Since the passage of a seismic wavelet implies local changes in fluid pressures and consequent fluid flow, the transport of seismo-acoustic energy through a rock is linked to fluid flow, and we may extend the electro-kinetic phenomenon to a seismo-electro-kinetic phenomenon. It is possible, in principle to perturb a layered earth with a seismic pulse, then to measure the resulting electrical signals as a function of offset. The so-called seismo-electric method relies on differences in the seismo-electro-kinetic coupling at interfaces in the subsurface and has recently been used to successfully image the vadose zone of a sand aquifer. Although several authors have recognised the potential of the method to provide direct access to rock properties such as porosity and permeability, little progress has been made. This is partly because the underlying physics of the seismo-electro-kinetic phenomena is not well described even though successful formulations for the DC regime have been available since Helmholz in 1879. However, recently there has been important progress in the AC formulation of the seismo-electro-kinetic phenomena that is necessary for use with the seismo-electric method and in the laboratory determination of the DC and AC coupling coefficients and associated zeta potential.

CONCLUSIONS

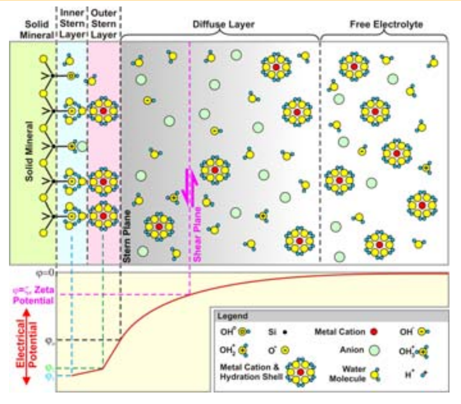
The major conclusions of this work are:

- This simplified approach to calculating the frequency-dependent coupling coefficients in porous media is valid for capillary tubes, bundles of capillary tubes of the same and different radii for different porosities.
- Neither the hydraulic or streaming potential coupling coefficients follow a Debye or a Cole and Cole type dispersion curve.
- However the hydraulic coupling coefficient follows a Debye type curve for frequencies lower than the characteristic dispersion frequency.
- For the hydraulic coupling coefficient each capillary size contributes at its characteristic frequency.
- However, the streaming potential coupling coefficient is controlled by the smallest tube radius because those have the greatest fluid velocity for a given imposed pressure difference, and the fact that there are many more tubes of the smaller size in a porous medium of a given porosity.

This study represents only the beginning of our modelling. Future plans include:

- Generalisation of the capillary model to porous media composed of grains using porosity-based power laws together with appropriate mixing laws (geometric mixing for randomly arranged pore radii), and
- Implementation of Hanai-Bruggeman mixing with the coupling coefficients.

ORIGIN OF (SEISMO)-ELECTRO-KINETIC PROPERTIES



A sketch of the structure of the electrical double/triple layer. The solid surface has exposed oxygens which react with water to give O^-, OH^o and OH^{2+} adsorption sites. Here there are 1 OH^o, 1 OH^{2+} and 3 O^- sites. At geological pHs the OH^- sites dominate strongly. The charged sites attract a loose following of water molecules arising because of their weak dipole moment. The negative sites attract cations from the bulk solution (which are surrounded by a hydration shell of water molecules). This is the adsorbed (Stern) layer in the EDL. Sometimes a distinction is made (as shown) between the internal and external Stern layers to make a three layer model (ETL). The bulk solution is locally depleted of cations; an effect which is weaker the further one gets from the Stern Plane (decreasing grey intensity) until unperturbed bulk solution is reached.

The bottom panel shows electrical potential as a function of distance away from the mineral surface. It increases from a negative value towards zero and is exponential in the diffuse layer

$$\varphi = \varphi_o \exp\{-\chi/\chi_d\}$$

where φ_o is the potential at the Stern plane, χ is the distance from the stern plane towards the bulk fluid, and χ_d is the Debye length. The shear plane represents the plane between water movement and stagnation. This is fundamental to electro-kinetic processes as fluid movement (e.g., upwards) will move more negative charges upwards than positive charges creating a charge separation and hence the streaming potential.

DC STREAMING POTENTIALS

The DC electro-kinetic coupling coefficient is given by the Helmholz-Smoluchowski equation:

$$C_s = \frac{\Delta V}{\Delta P} = \frac{\varepsilon\zeta}{\eta\sigma}$$

ΔV	Streaming potential (mV)	ΔP	Applied fluid pressure difference (Pa)
ε	Fluid dielectric permittivity (F/m)	η	Fluid viscosity (Pa.s)
σ	Fluid conductivity (S/m)	ζ	Zeta potential (mV)

Ideally, the fluid conductivity includes a contribution related to the surface conduction where Σ_s is the surface conductance and Λ is a length characteristic of the pore space.

$$\sigma = \sigma_f + \frac{2\Sigma_s}{\Lambda}$$

AC STREAMING POTENTIAL AND HYDRAULIC COUPLING COEFFICIENTS

The AC streaming potential and hydraulic coupling coefficients have been calculated for a bundle of capillary tubes starting from the Navier-Stokes equation with a sinusoidal driving pressure (Packard, 1953)

$$\rho\frac{\partial\overline{\upsilon}(r,\omega)}{\partial t}\exp(-i\omega t) = -\nabla P(\omega)\exp(-i\omega t) + \eta\nabla^2\overline{\upsilon}(r,\omega)\exp(-i\omega t)$$

This provides a solution for the hydraulic coupling coefficient in terms of Bessel functions for the average flow $\upsilon(\omega)$ in a capillary of length *l* and radius *a*.

$$C_h = \frac{\upsilon(\omega)}{\Delta P(\omega)} = \frac{1}{\eta l k^2}\left[\frac{2}{ka}\frac{J_1(ka)}{J_0(ka)} - 1\right] \quad \text{where} \quad k = \sqrt{\frac{-i\omega\rho}{\eta}}$$

(Reppert et al., 2001)

Similarly a solution in terms of Bessel functions for the streaming potential coupling coefficient in a capillary of length *l* and radius *a*.

$$C_s = \frac{\Delta V(\omega)}{\Delta P(\omega)} = \left[\frac{\varepsilon\zeta}{\eta\sigma}\right]\left[\frac{-2}{ka}\frac{J_1(ka)}{J_0(ka)}\right]$$

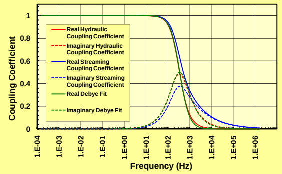
Reppert et al. (2001) have simplified C_s to give.

$$C_s(\omega) = \left[\frac{\varepsilon\zeta}{\eta\sigma}\right]\left[1 - \frac{2}{a}\sqrt{\frac{\eta}{\omega\rho}}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)\right]$$

compared with the value found by Pride (1994).

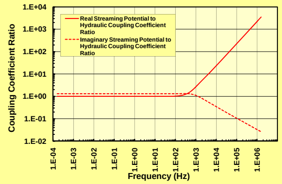
$$C_s(\omega) = \left[\frac{\varepsilon\zeta}{\eta\sigma}\right]\left[1 - i\frac{\omega}{\omega_t}\frac{m}{4}\left(1 - \frac{2d}{\Lambda}\right)^2\left(1 - i^{3/2}d\sqrt{\frac{\omega\rho}{\eta}}\right)^2\right]^{-1/2}$$

We have taken the equations for the coupling coefficients (in their Bessel function form) and used them in modelling to represent each coupling coefficient as a function of frequency for packs of different capillary tube radii and porous media composed of spherical and sub-spherical grains. In each case the coupling coefficients are normalised for ease of comparison.

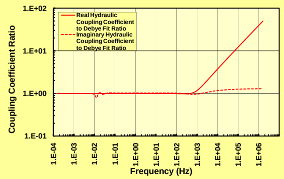


Comparison of the hydraulic and streaming potential coupling coefficients (real and imaginary) as a function of frequency.

The streaming potential coupling coefficient is clearly asymmetric (i.e., non-Debye, non-Cole and Cole). The hydraulic coupling coefficient is similar to a Debye function. The best Debye fit is shown.



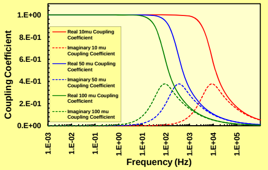
The ratio of the streaming potential coupling coefficient to the hydraulic coupling coefficient (i.e., the streaming potential to fluid velocity coupling coefficient). The real part shows equality lower than the central frequency of the dispersion, but diverges at higher frequencies due to the asymmetry in the streaming potential coupling coefficient, with the normalised streaming potential taking values higher than that of the normalised hydraulic coupling coefficient. The imaginary part shows a constant ratio of about 1.3 for frequencies below the dispersion frequency and with the normalised streaming potential taking values lower than that of the normalised hydraulic coupling coefficient.



The ratio of the hydraulic coupling coefficient to the best Debye-type (i.e., the fit. While the fit is good below the dispersion frequency, the Debye fit underestimates the hydraulic coupling coefficient at higher frequencies for both the real and the imaginary parts.

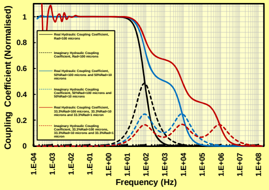
The streaming potential coupling coefficients (real and imaginary) as a function of frequency and of pore size.

Increasing pore size from 10 microns to 100 microns shifts the dispersion to lower frequencies without changing the shape of the dispersion.



The real and hydraulic coupling coefficients for a porous medium composed of 100% capillary tubes with a 100 micron radius and a porosity of 0.2 (black) compared with media composed of 50% 100 micron radius, 50% 10 micron radius tubes, same porosity, and one with 33.3% each of 100, 10 and 1 micron tubes with the same porosity.

Each capillary size contributes to the frequency dependent coupling coefficient at its characteristic frequency.



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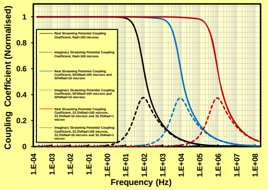
The coupling coefficient is controlled by the smallest tube radius because those have the greatest fluid velocity for a given pressure and there are tubes of that size in the porous medium.



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