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A Generalized Archie's law for n phases

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CONNECTEDNESS

The connectedness of a porous medium G was defined by Glover (2009) as the physical availability of pathways for transport, and mathematically as the ratio of the conductivity of the rock to that of the pore fluid (i.e., inverse of the formation factor)

$$G \equiv \frac{\sigma}{\sigma_f} = \frac{1}{F} = \phi^m$$

The connectedness G of a given phase is a physical measure of the availability of pathways for conduction through that phase. It is the ratio of the measured conductivity to the maximum conductivity possible with that phase (i.e., when that phase occupies the whole sample). This implies that the connectedness of a sample composed solely of a single phase is unity.

By contrast, the connectivity is defined as the measure of how the pore space is arranged. The connectivity is given by

$$\chi = \phi^{m-1}$$

and depends upon the porosity ϕ and the cementation exponent m. It should be noted that the connectedness is also given by

$$G = \phi\chi$$

and it is clear that the connectedness depends both upon the amount of pore space (given by the porosity) and the arrangement of that pore space (given by the connectivity).

Conclusion: The connectedness is a fundamental and useful property of a phase in a multidimensional mixture of n phases

COMPARISON WITH EXISTING MODELS

Examination of the exact solution of the generalized Archie's law for n phases shows it to be formally the same as (i) Archie's traditional law for a single phase, and (ii) the modified Archie's law for two phases (Glover et al., 2000).

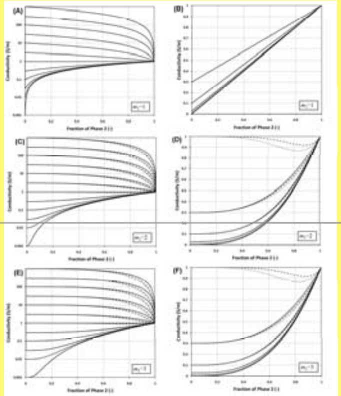


Figure 1. The variation of total conductivity as a function of the volume fraction of Phase 2 for the generalized Archie's law applied to a two-phase system for three different values of m_2. In all cases the conductivity of Phase 2 is constant and equal to 1 S/m. Calculations have been carried out for 12 different values of the conductivity of Phase 1, which can be read on the y axis when ϕ_2=0. Parts (A), (C) and (E) show the full variation on semi-log axes. Parts (B), (D) and (E) show the same data on linear axes in order to better show the variation when the conductivity is less than 1 S/m. Solid lines represent both the exact solution of the generalized Archie's law (Eq. 1 ➡) and the solution of the modified Archie's law for two phases (Glover et al., 2000), which are collinear. The dotted lines represent the first order solution of the generalized Archie's law (Eq. 2 ➡) , and the dashed lines represent its second order solution (equation not shown on poster).

$$\sigma = \sum_i^n \sigma_i \phi_i^{m_i} \rightarrow \sigma_f \phi^m + \sigma_m (1-\phi)^p \quad \text{for two phases (Glover et al., 2000)}$$

$$\rightarrow \sigma_f \phi^m \quad \text{for one phase (Archie, 1942)}$$

Conclusion: The generalised form is compatible with existing forms

PRINCIPLE OF CONSERVATION OF CONNECTEDNESS

The classical result is that the sum of all volume fractions is unity.

$$\sum_{i=1}^{n} \phi_i = 1$$

Now we consider an analogous result for the connectedness. Intuitively, it seems reasonable that as one or more phases increase their connectedness, other phases must lose connectedness. This idea leads to the hypothesis that there is a fixed maximum amount of connectedness possible in a three-dimensional sample. It is possible to distribute it between whichever phases are present in an infinite number of ways, but the total connectedness must not exceed some maximum value that is defined by the topology of the three-dimensional space.

Conclusion: The sum of all connectednesses is unity

$$\sum_i^n \phi_i^{m_i} = 1 \qquad \sum_i^n G_i = 1$$

The sum of all connectednesses is unity

$$\sum_i^n \phi_i^{m_i} = \sum_i^n G_i = 1$$

The sum of all connectednesses is unity

$$\sum_{i=1}^{n} \phi_i = 1$$

The sum of all volume fractions is unity

$$\sigma = \sum_i^n \sigma_i \phi_i^{m_i}$$

The generalised Archie's law

PHASE EXPONENTS

The phase exponent of the j^{th} phase can be calculated using

$$m_j = \log\left(1 - \sum_{i \neq j} \phi_i^{m_i}\right) \Big/ \log\left(1 - \sum_{i \neq j} \phi_i\right) \quad \text{(Eq. 1)}$$

The phase exponent of the jth phase can also be calculated using a first order approximation by the equation

$$m_j = \sum_{i \neq j} G_i \Big/ \sum_{i \neq j} \phi_i \quad \text{(Eq. 2)}$$

Furthermore a n -1 subset of phases has the property that $\sum_{i=1}^{n-1} S_i^{(n_i - m_i)} = 1$

where the saturation of each phase is $S_i = \phi_i \Big/ \sum_{i=1}^{n-1} \phi_i$ and n_i is the saturation exponent of each phase.

Conclusion: All phase exponents are linked to each other and to their relevant saturation exponents.

PARTIAL VALIDATION BY MODELLING

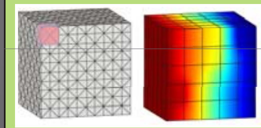


Figure 6. Left-hand side, geometry of the numerical model (here a 5×5×5 model is shown for simplicity. The real size was 20×20×20) and the tetragonal grid used in the numerical modeling. Right-hand side, an example of a numerical solution of electrical potential across the cube (red, left hand side, 10V; blue, right-hand side, 0V).

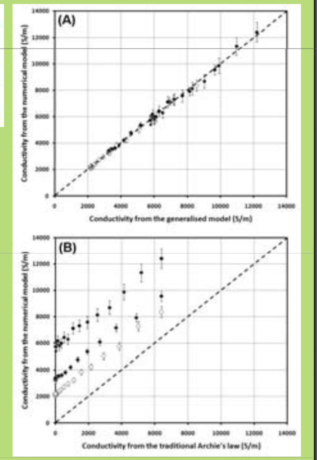
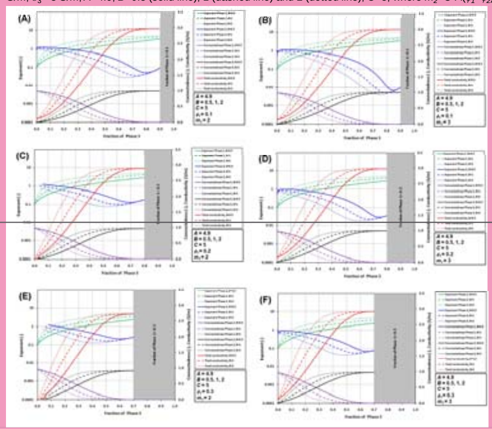


Figure 7. Results of numerical modeling of a four phase system. (A) Conductivity from the numerical modeling as a function of that predicted from the generalized Archie's law for the three suites of modeling results. (B) Conductivity from the numerical modeling as a function of that predicted from the traditional Archie's law for the 3 suites of modeling results. Note that because the conductivity of Phase 4 is so low, this diagram is also a plot of the conductivity from the numerical modeling as a function of that predicted from the modified Archie's law for 2 phases (Glover et al., 2000a).

Conclusion: Numerical modelling confirms that the generalised Law models a 4 phase model well.

APPLICATION TO 3 PHASES

Figure 5. The generalized Archie's law for three phases: the effect of changing the volume fraction and exponent of Phase 1. The variation of connectednesses and exponents of Phases 2 and 3 together with the total resulting conductivity are shown as a function of the volume fraction of Phase 3. Green = m_2, blue = m_3, purple = G_2 , black = G_3, red = total conductivity (S/m). Common values: σ_1= 0.02 S/m, σ_2= 0.1 S/m, σ_3= 3 S/m, A=4.9, B=0.5 (solid line), 1 (dashed line) and 2 (dotted line), C=5, where m_2=C-A(ϕ_1+ϕ_2)^B.



Conclusion: It is necessary to solve $m_i(\phi_i)$ for all phases in order to have a full understanding of any n phase system.

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