

Permeability models of porous media: Characteristic length scales, scaling constants and time-dependent electrokinetic coupling

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INTRODUCTION

Four important models that describe the fluid permeability of geological porous media and that are derived from different physical approaches have been rewritten in a generic form that implies a characteristic scale length L and scaling constant c for each model (G is the connectedness of the rock).

$$k_j = \frac{G L_j^2}{c_j}$$

The four models have been compared theoretically and using experimental data from 22 bead packs and 188 rock cores from a sand-shale sequence in the North Sea.

The **Kozeny-Carman** model does not perform well because it takes no account of the connectedness of the pore network, and should no longer be used. The other three models (**Schwartz, Sen and Johnson (SSJ)**, **Katz and Thompson (KT)** and the so-called **RGPZ**) all performed well when used with their respective length scales and scaling constants.

Surprisingly, we have found that the **SSJ** and **KT** models are extremely similar, such that their characteristic scale lengths and scaling constants are almost identical even though they are derived using extremely different approaches; the **SSJ** model by weighting the **Kozeny-Carman** model using the local electric field, the **KT** model using entry radii from fluid imbibition measurements.

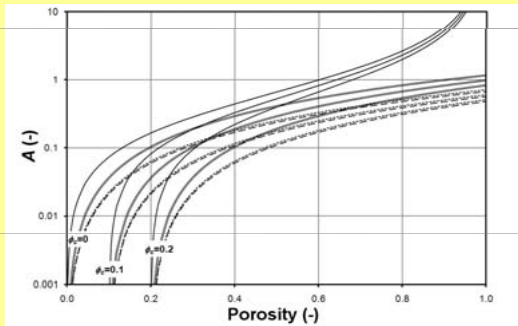
Use of these models with AC electrokinetic theory has also allowed us to show that these scaling constants are also related to the a value in the **RGPZ** model and the m^* value in time-dependent electrokinetic theory, and then derive a relationship between the electrokinetic transition frequency and the **RGPZ** scale length, which we have validated using experimental data. The practical implication of this work for permeability prediction is that the **Katz and Thompson** model should be used when fluid imbibition data is available, while the **RGPZ** model should be used when electrical data is available.

THE A COEFFICIENT

Figure 1. The A coefficient is the porosity dependent part of the characteristic scale length L according to the general equation on the right.

$$L_j = A_j r_{grain}$$

It is different for each model, as shown. **Kozeny-Carman** – solid lines, **SSJ** – long dashed lines, **KT** – short dashed lines, **RGPZ** – double lines. In each case there is a set of three curves for values of critical porosity $\phi_c = 0, 0.1, \text{ and } 0.2$. The **Kozeny-Carman** model is independent of cementation exponent, $m=1.5$ has been used for the other models.



SCALING CONSTANTS

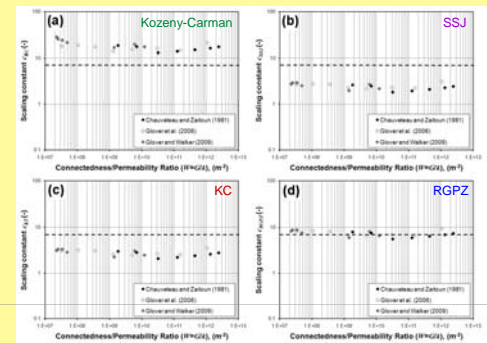


Figure 2. Scaling constants calculated from the permeability models of (a) **Kozeny-Carman**, (b) **Schwartz, Sen and Johnson**, (c) **Katz and Thompson**, and (d) **Revil, Glover, Pezard and Zamora**, using data from 22 glass bead pack experiments (Chauveteau and Zaitoun (1981), Glover et al. (2006) and Glover and Walker (2009)) as a function of the connectivity-permeability ratio W as defined in this work. The horizontal dashed line represents the value $c_0=8$.

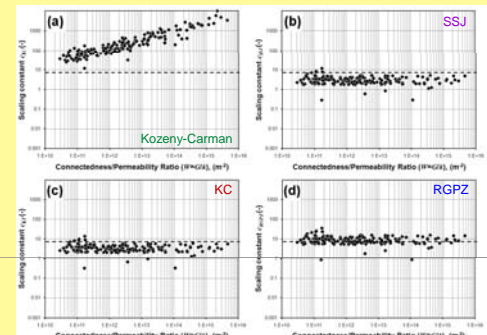


Figure 4. Scaling constants calculated from the permeability models of (a) **Kozeny-Carman**, (b) **Schwartz, Sen and Johnson**, (c) **Katz and Thompson**, and (d) **Revil, Glover, Pezard and Zamora**, using data from 188 rock cores from a sand-shale sequence of the U.K. North Sea (Glover and Walker (2009)) as a function of the connectivity-permeability ratio W as defined in this work. The horizontal dashed line represents the value $c_0=8$.

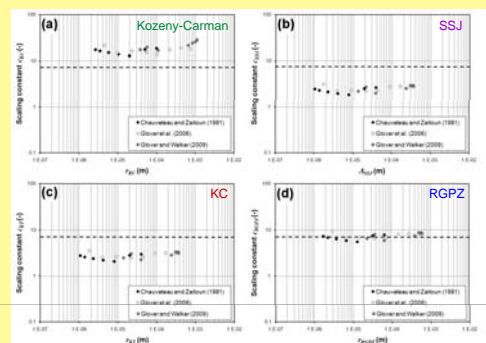


Figure 3. Scaling constants calculated from the permeability models of (a) **Kozeny-Carman**, (b) **Schwartz, Sen and Johnson**, (c) **Katz and Thompson**, and (d) **Revil, Glover, Pezard and Zamora**, using data from 22 glass bead pack experiments (Chauveteau and Zaitoun (1981), Glover et al. (2006) and Glover and Walker (2009)) as a function of the relevant length scale r_i for each model. The horizontal dashed line represents the value $c_0=8$.

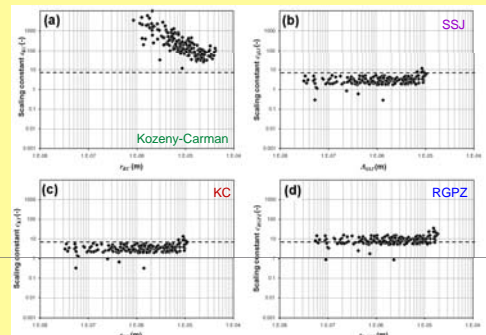


Figure 5. Scaling constants calculated from the permeability models of (a) **Kozeny-Carman**, (b) **Schwartz, Sen and Johnson**, (c) **Katz and Thompson**, and (d) **Revil, Glover, Pezard and Zamora**, using data from 188 rock cores from a sand-shale sequence of the U.K. North Sea (Glover and Walker (2009)) as a function of the relevant length scale r_i for each model. The horizontal dashed line represents the value $c_0=8$.

$$r_{KC} = A_{KC} r_{grain} = \frac{2(\phi - \phi_c)}{3(1 - \phi)} r_{grain}$$

$$r_{SSJ} = A_{SSJ} r_{grain} = \frac{1}{m(\phi - \phi_c)^m} r_{grain}$$

$$r_{KT} = A_{KT} r_{grain} = \frac{\sqrt{3}}{R_i m(\phi - \phi_c)^m} r_{grain}$$

$$r_{RGPZ} = A_{RGPZ} r_{grain} = \frac{\sqrt{3}}{m(\phi - \phi_c)^m} r_{grain}$$

ELECTRO-KINETIC TRANSITION FREQUENCY

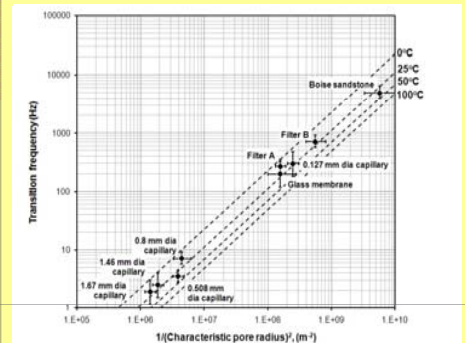


Figure 6. The electro-kinetic transition frequency as a function of the inverse square characteristic pore size. The dashed lines represent the result of the equation at 4 different temperatures for water as the pore fluid and with the respective densities and viscosities (For $T=0^\circ\text{C}$, $\eta = 1.79 \cdot 10^{-3}$ Pa.s and $\rho = 10^3$ kg/m³; for $T=25^\circ\text{C}$, $\eta = 0.89 \cdot 10^{-3}$ Pa.s and $\rho = 997$ kg/m³; for $T=50^\circ\text{C}$, $\eta = 0.547 \cdot 10^{-3}$ Pa.s and $\rho = 988$ kg/m³; for $T=100^\circ\text{C}$, $\eta = 0.282 \cdot 10^{-3}$ Pa.s and $\rho = 589.67$ kg/m³ from Lide (2009).) Measured data from Reppert et al. (2001), Reppert (2000), Packard (1951), Sears and Groves (1977), and Cooke (1955).

CONCLUSIONS

- Four important models that describe the fluid permeability on porous media have been compared and been found to follow the same generic form even though they are derived from very different physics.
- The difference between the models depends upon which scale length they use, and which scaling constant is then employed to validate the model.
- We have studied the scale lengths and scaling constants both theoretically and using experimental data.
- The **Kozeny-Carman** model did not perform well, as has been noted before by many authors (e.g. Scheidegger, 1974; Bernabé, 1995), and should no longer be used. The problem with this model is that it takes no account of the connectedness of the pore network.
- The **Schwartz, Sen, Johnson (SSJ)**, the **Katz and Thompson (KT)** and the **RGPZ** models all performed well when used with their respective length scales and scaling constants.
- It was noted that the **SSJ** and **KT** models were extremely similar such that $c_{SSJ} \approx c_{KT}$ and $A_{SSJ} \approx A_{KT}$ despite the disparity in their physical derivation.
- Comparison of the models with experimental data from 22 bead packs and 188 rock cores from a sand-shale sequence in the UK sector of the North Sea has provided values for the scaling constants for each model, with $a = m^* = c_j = c_{SSJ} = c_{KC} \approx 8/3$ and $c_{RGPZ} \approx 8$.
- Use of time-dependent electrokinetic theory allows us to also equate some of the scaling constants to the m value used by Pride (1994), the a value used by Glover et al. (2006) and the c_j value used by Bernabé (1995): $a = m^* = c_j = c_{SSJ} \approx c_{KC}$ and $a = m^* = c_j = c_{RGPZ} \approx 8$.
- We have derived a relationship between the electrokinetic transition frequency and the **RGPZ** scale length, and validated it using experimental data from ceramic filters, glass membranes, capillary tubes and one sandstone.